CHAPTER 6

Improving methods and models for artillery combat control

Pavlo Gultsov
Oksana Maksymova
Yevhenii Dobrynin

Abstract

This chapter deals with methodological approaches to estimating the effectiveness of the combat employment of artillery units considering minimization of the loss of combat capability. The definition of combat capability is proposed, which is seen as the ability of a unit to perform assigned tasks under specified operating and external conditions. The influence of external factors and loads on the state of the unit and on the intensity of failure of its elements is analyzed. A stochastic model of the combat operation of an artillery battery is presented. The model is based on a discrete Markov chain structure which allows to estimate time and dynamic characteristics, including the recovery rate, the probability of destructing a target and the average firing time. Criteria for an effective shot are defined. The use of a combined criterion to estimate combat effectiveness is justified. The proposed approaches provide a scientific basis to predict combat capability, optimize control processes and decision-making in automated systems for estimating artillery fire effectiveness.

Simulation results demonstrate that the proposed approach allows maintaining the combat capability at a level of 0.78–0.81 for a 10-shot mission, while the ratio between optimal and worst mission structures reaches values from 4 to 10.

Keywords

Combat capability, artillery unit, stochastic modeling, firing effectiveness, Markov model, combat employment control.

6.1 Introduction

This section describes the problem of improving the control efficiency of the combat employment of an artillery unit in conditions of counter-battery engagements

 DOI: 10.21303/978-9908-8450-1-2.ch6

with existing disturbances connected with changes of firing positions and with the influence of the opposing side. Earlier results aiming at minimizing the loss of combat capability of an artillery unit and reducing the time needed to complete a fire mission are summarized and extended [1, 2].

It was used an approach to form a generalized criterion for estimating the combat employment of an artillery unit which accounts for firing accuracy, time indicators and ammunition expenditure [2, 3]. Stochastic simulation based on Markov models is used to analyze the combat process. The model for computing the current probabilistic state of combat capability of an artillery unit was also improved taking into account sudden failures and failures caused by the loss of combat capability, as well as the method for forming states when the opposing side fires to destroy the current firing position and when this position is changed.

6.2 General provisions on an effective shot

In this section, it is possible to assume that the combat capability of an artillery unit is determined by quantitative parameters. They are the muzzle velocity of the projectile, the speed of transporting the unit between firing positions and the energy characteristic of the artillery charge which is evaluated by the strength of the powder.

An artillery unit is considered combat-capable when the muzzle velocity matches the tabulated value, the transportation speed meets the regulatory norms and the powder strength corresponds to its grade during verification [1, 4].

Forming the generalized criterion for estimating the combat employment of an artillery unit, several simplifications are adopted.

Regular firing errors are connected with topological features of the current firing position and the routes of transporting a unit between positions. They are not detailed further.

The location of the target and meteorological conditions are accounted for during firing preparation. The model considers only changes in properties that directly influence combat capability: muzzle velocity, transportation speed, and powder strength in the charge [1, 4].

Some provisions of the modern decision-making theory are used to analyze the gun firing process [2].

To estimate the effectiveness of the combat employment of an artillery unit, the concept of a generalized criterion is used. It is formed by aggregating partial indicators. The partial indicators are normalized to their maximum values.

Accuracy indicator

$$Crit_{accuracy} = \frac{CEP_{fire}}{CEP_D}, \tag{6.1}$$

task execution time indicator

$$Crit_{time} = \frac{time_{fire}}{time_{fire}^{limit}}, \tag{6.2}$$

ammunition expenditure indicator

$$Crit_{ammo} = \frac{n_{shoot}^{non\_eff}}{n_{shoot}^{total}}, \tag{6.3}$$

where CEP_{fire} – the circular error probable (CEP) of firing; CEP_D – the CEP for range D; $time_{fire}$ – the firing time of the artillery unit to hit the target; $time_{fire}^{limit}$ – the time limit of firing at the current position; $n_{shoot}^{non\_eff}$ – the number of ineffective shots; n_{shoot}^{total} – the total number of shots.

For guns of caliber above 122 mm CEP_D can be assumed to be about 1% D (that is, $R = 0.01D$). Field data show that the time indicator usually does not exceed 0.75. In real conditions, it is 0.6–0.7 of the full firing time [1].

Accounting for these indicators, the generalized criterion is formed as the distance to the ideal point using the L_2 norm

$$Crit_0^{ideal\_point} = \sqrt{\sum_{i=1}^{n} (Crit_i - Crit_i^{ideal\_point})^2}, i = 1,...,n, CEP_D, \tag{6.4}$$

where $Crit_i^{ideal\_point}$ – the value of partial indicators at the ideal point in an n-dimensional space, where all indicators take their minimum values; $Crit_i \in \{Crit_{accuracy}, Crit_{time}, Crit_{ammo}\}$ – the partial indicators of the generalized criterion.

A shot is considered ineffective when the muzzle velocity satisfies $v_0 < 0.95 v_{table\_fire}$. A shot is effective when the artillery projectile hits a circle of radius $R = 1\% \, D$ with probability at least 0.5 [1].

6.3 Modeling the combat operation of artillery unit

In this section, the main criterion for estimating the combat operation of an artillery unit is combat capability. This indicator describes the ability of the unit to perform a combat task with effectiveness not lower than a specified level.

It decreases under the influence of external factors, including the actions of opposing units [5–7].

Combat capability is regarded as the degree of resistance of an artillery unit to external impacts. It is a relative indicator in the range from 0 to 1. It reflects the current state of the unit during a combat task [5].

Combat capability includes resistance to mechanical, thermal and electrical loads, vibration, shocks and environmental impacts. It also includes the ability to counteract the opposing side that aim to reduce the effectiveness of combat employment.

In practice, the combat capability state is estimated by analyzing the operability of the elements of the artillery unit under specified loads. This makes it possible to determine the failure intensity for the corresponding operating modes [8, 9].

Changing the load conditions and repeated estimate of operability will allow to determine how failure intensity depends on the magnitude of impacts. In this context, failure intensity can serve as a general quantitative measure of the combat capability of an artillery unit [10].

This interpretation makes it possible to quantitatively evaluate the impact of operational and enemy-induced factors on combat capability and to directly compare different combat scenarios using a unified numerical indicator.

To describe combat capability in military practice, probabilistic models are widely used. In such models, the state of an artillery unit is determined by two components. They are sudden failures and failures due to wear of unit elements [11, 12]. Sudden failures are usually modeled by an exponential law. In this case, combat capability is interpreted as the probability of failure-free operation and is defined as

$$P(t)=e^{-\lambda t}, \tag{6.5}$$

where λ – the intensity of sudden failures. It affects the muzzle velocity of the projectile, the transportation speed between firing positions and the energy characteristic of the artillery charge (powder strength).

In the general case

$$\lambda=-\frac{1}{P(t)}\frac{dP(t)}{dt}. \tag{6.6}$$

The following relation is important for any distribution law

$$\lambda=\frac{f(t)}{P(t)}, \text{ where } f(t)=-\frac{dP(t)}{dt}, \tag{6.7}$$

where $f(t)$ – the probability frequency of failures.

In normal combat operation, the failure intensity is constant for the exponential law. When combat capability is lost, the failure intensity begins to grow. Failures due to the loss of combat capability are added to sudden failures. In engineering practice, they are modeled by a normal distribution

$$P_w(t)=\frac{1}{\sigma\sqrt{2\pi}}\int_t^{\infty} e^{-(t-M)^2/2\sigma^2}\,dt, \tag{6.8}$$

where M – the mean value of the combat capability of the artillery unit.

The standard deviation from the mean combat capability is defined as

$$\sigma=\sqrt{\frac{\sum(t-M)^2}{N}}, \tag{6.9}$$

where N – the number of failures during time t.

The joint probability of the combat capability of an artillery unit, taking into account all types of failures over the period from $t_0=0$ to t is defined as

$$P(t)=e^{-\lambda t}P_w(t). \tag{6.10}$$

In the case of a partial loss of combat capability, the joint probability is

$$P(t)=e^{-\lambda t}\frac{P_w(t_0+t)}{P_w(t_0)}. \tag{6.11}$$

Expression (6.11) allows one to estimate the combat capability of an artillery unit at any time. However, it has drawbacks for this study. Unregulated operation of a technical artillery unit leads to increased wear. According to field observations, failure intensity strongly depends on the quality of operation of the artillery unit according to its technical requirements [13, 14].

When the load exceeds the nominal level, a noticeable increase in the number of failures is observed. At the same time, the proposed expression does not account for a sudden decrease in combat capability when the artillery unit is hit by the opposing side. In contrast, failure intensity decreases when the load becomes lower than the nominal level [2, 3].

As these factors primarily affect failure intensity, the following expression is used for reliability modeling

$$P(t)=e^{-\int_T^{T+t}\lambda dt}=e^{-\lambda_s t+\int_T^{T+t}\lambda_w dt}, \tag{6.12}$$

where λ_s – the intensity of sudden failures, and λ_w – the intensity of failures caused by the loss of combat capability.

To determine the failure intensity due to the loss of combat capability, the following approach is used. From (6.7), in the general case, failure intensity is defined as $\lambda = f(t)/P(t)$.

In the simulation, the combat capability of the artillery unit changes by Δt at each iteration of the current operating time. Then, let's use expression (6.12) to compute the current value of the failure intensity due to the loss of combat capability.

If, at the current simulation iteration, the unit is under a forced impact that speeds up the loss of combat capability, an additional value Δt_w is added to the operating time. This value corresponds to the degree of combat capability reduction. The expression $P(t) = e^{-\lambda t}$ describes the change of combat capability over time when only sudden failures are present. The expression $P(t) = P_w(T+t)/P_w(t)$ describes the change of combat capability during combat operation without considering destruction by the opposing side [1, 2].

6.4 Stochastic model of firing of an artillery unit

The artillery battery model [1] makes it possible to study combat-operation conditions for different types of artillery weapons – from mortars to heavy self-propelled systems. This is achieved by changing the tuning parameters. The model is given as a non-reduced positive recurrent Markov chain. Within it, the artillery battery is represented as a single whole. This enables comparison of weapon systems and combat-operation algorithms.

The state-space equations are solved as limiting distributions, which ID determined by the properties of the Markov chain. The simulation result is the time interval during which the battery remains in a state that provides fire support [1, 15].

The logical structure of the model of artillery battery combat-operation is based on practical experience of its use. While occupying a new firing position, the battery usually remains unnoticed by the opposing side. A main detection tool is a passive acoustic counter-battery radar.

Detection of the first shot by the radar makes it possible to locate the battery with a certain error and a given probability [13, 16].

The detection probabilities for a single shot and for a salvo are assumed to be the same. The overall detection probability depends on the number of shots fired before detection.

The flow of targets of different types over time is modeled as a homogeneous Poisson process. For each target type, the artillery battery performs a predefined number of shots.

The battery changes its firing position after completing a specified number of shots, taking into account the available information about possible detection. If such information is received, the battery leaves the firing position with high probability before the opposing side begins an attack.

While moving between firing positions, the artillery battery does not provide fire support. If the opposing side attacks the current firing position, the battery suffers combat losses and damage. This can cause a temporary or irreversible loss of the ability to perform combat tasks.

With this combat logic described above, the recurrent Markov chain for the artillery battery includes the following states. Each state can reduce the current combat capability to a different extent when the corresponding events occur [1]:

– is at a firing position and provides fire support; the battery is not detected by the opposing side;

– is at a firing position and provides fire support; the battery is detected by the opposing side but has no information about the detection;

– is at a firing position, does not provide fire support, is detected by the opposing side, has information about the detection, and prepares to change the firing position;

– is moving to a new firing position and does not provide fire support;

– cannot provide fire support due to ammunition depletion or a complete loss of combat capability.

For simulating mission execution, a set of input data is formed. It includes parameters that describe detection and engagement conditions, such as the number of targets, target detection rate, the number of shots needed to hit one target and the probability of battery detection during firing. Another group includes time and dynamic characteristics of combat operation. They are the average firing time before an opposing attack after detection, the battery recovery rate after an attack, the speed of movement between firing positions, the probability of an attack during movement, and the decision-making and exit time from a firing position [1, 9].

The process of changing the combat employment of the artillery battery is represented as a graph, shown in **Fig. 6.1**. The vertices of the graph correspond to current battery states, and the edges describe transitions between these states.

The initial state S_0 defines the readiness of the battery for combat employment on condition of n guns availability for simultaneous firing. The following states S_i ($i=1,...,n$) describe a shot fired simultaneously by i guns. The edge weight l_{ij} is the probability of a Markov process transition from state S_i to state S_j. It is denoted as $p_{ij}, i, j = 0,...,n$.

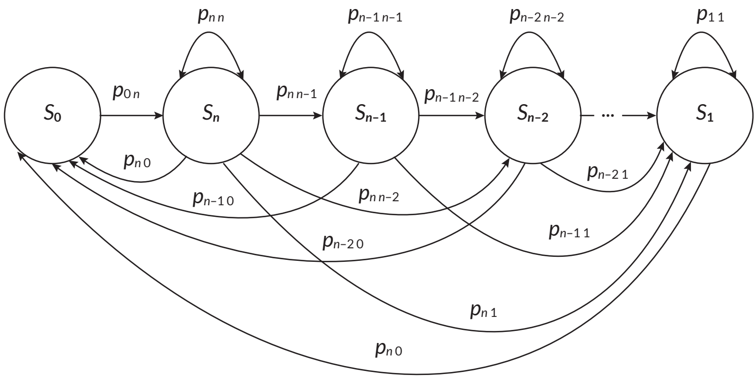

Fig. 6.1 Markov graph of states and transition probabilities

As the states are random, the process can be considered Markov. The stochastic model of automated control also accounts for disturbances. A characteristic disturbance is the difference between the current and the initial muzzle velocities of the projectile, Δv_0. It is caused by chamber wear $\Delta v_0^{\lambda\_chamb}$, barrel wear Δv_0^{barrel} and uncertainty in the charge energy, Δv_0^{charge}. If all three types of disturbances occur simultaneously, the shot effectiveness can be considered an infinitesimal value.

As the disturbances are random, let's consider the probability that the corresponding factor does not affect the shot result:

$$p_1 = p(\Delta v_0^{\lambda\_chamb}),$$

$$p_2 = p(\Delta v_0^{barrel}),$$

$$p_3 = p(\Delta v_0^{charge}). \tag{6.13}$$

Then the probability of the presence of the corresponding disturbance factor is:

$$p_1 = 1 - p(\Delta v_0^{\lambda\_chamb}),$$

$$p_2 = 1 - p(\Delta v_0^{barrel}),$$

$$p_3 = 1 - p(\Delta v_0^{charge}). \tag{6.14}$$

Thus, the transition matrix between states in the Markov modeling has the form:

$$P=\begin{Vmatrix} 0 & p_{0\,n} & 0 & 0 & \cdots & 0 \\ p_{n\,0} & p_{n\,n} & p_{n\,n-1} & p_{n\,n-2} & \cdots & p_{n\,1} \\ p_{n-1\,0} & 0 & p_{n-1\,n-1} & p_{n-1\,n-2} & \cdots & p_{n-1\,1} \\ p_{n-2\,0} & 0 & 0 & p_{n-2\,n-2} & \cdots & p_{n-2\,1} \\ \cdots & \cdots & \cdots & \cdots & \cdots & \cdots \\ p_{1\,0} & 0 & 0 & 0 & \cdots & p_{1\,1} \end{Vmatrix}. \tag{6.15}$$

The goal of the Markov model is to determine the number of shots for which the probability of hitting the target is not less than a specified value.

The transition matrix is filled as follows. The first row contains a single non-zero element $p_{0n}=1$. This means that a transition from the initial state to state S_n is possible only when all n guns are ready for combat employment.

The other elements are computed using row i as an example:

$$i=1,...,n;$$

$$p_{i0}=1-\sum_{j=1}^{n}p_{ij};\ p_{ij}=\begin{cases} C_i^j p^j q^{i-j},\ j\le i; \\ 0,\ j>i, \end{cases} \tag{6.16}$$

where p is the probability of an effective shot.

The sum of elements in each row equals 1. Therefore, the matrix satisfies the stochasticity condition.

Thus, the control process can be interpreted as a sequence of states. Each state corresponds to the ability to perform an effective shot by a certain number of artillery units.

If an ineffective shot occurs at the current simulation step, the corresponding unit is excluded from the subsequent mission execution process. Firing continues using a smaller number of units.

A limiting state is possible where all units perform ineffective shots. In this case, according to the adopted conditions, the combat task becomes impossible.

Using model (6.16), it is possible to compute the probability of effective shots after M cycles of combat employment of the artillery battery

$$P_M=(P_0)^M, \tag{6.17}$$

where P_0 – the transition probability matrix after the first firing cycle.

Let's take the number of effective shots as the expected value of a random variable that describes the number of guns that perform an effective shot. For this purpose, it is possible to introduce the state vector sv_i. Its components indicate which guns participate in an effective shot

$$sv_i = \left[sv_j = 1,\ j = i;\ sv_j = 0,\ j \neq i;\ j = 1,\dots,n \right]^{\mathrm{T}}. \quad (6.18)$$

An example of using the Markov model to evaluate the combat employment results for a battery of 6 guns is given below. The probability of an effective shot when the corresponding disturbances are absent is: $p_1 = 0.95$ (no chamber wear), $p_2 = 0.95$ (no barrel wear), and $p_3 = 0.9$ (no uncertainty of charge energy). Then the probability of an effective shot in the absence of all three types of disturbances is $p = p_1 p_2 p_3 = 0.81225$. This value illustrates that even moderate degradations of individual subsystems lead to a significant cumulative reduction in firing effectiveness, which is explicitly captured by the proposed Markov-based model.

Table 6.1 shows the results of computing the transition matrix between states according to the Markov model (6.16).

Table 6.1 Transition probability matrix

State	S_0	S_6	S_5	S_4	S_3	S_2	S_1
S_0	0	1	0	0	0	0	0
S_6	4E-05	0.2872	0.3983	0.2302	0.0709	0.0123	0.0011
S_5	0.0002	0	0.3536	0.4086	0.1889	0.0437	0.0051
S_4	0.0012	0	0	0.4353	0.4025	0.1395	0.0215
S_3	0.4365	0	0	0	0.4025	0.1395	0.0215
S_2	0.0353	0	0	0	0	0.6598	0.305
S_1	0.188	0	0	0	0	0	0.812

The obtained transition probabilities allow estimating not only the expected number of effective shots, but also the rate of degradation of the artillery battery as guns are sequentially excluded from mission execution.

6.5 Model of combat employment of an artillery unit

Let's consider a model of the control problem for the combat operation of an artillery unit in the following setting. Artillery unit AU1 must perform a combat task.

The task is to destroy a stationary target with n shots on condition of full firing preparation with the probability of firing position change.

To solve the problem, let's introduce assumptions required for modeling. A finite number of firing positions and a given number of shots n required to engage the target is assumed. The minimum number of shots from one firing position equals one. Thus, at least one shot must be fired from each current position.

A change of firing position does not allow returning to previous positions. Movement from one position to a neighboring one is carried out sequentially along one of s roads. The roads differ in surface quality and in the probability that the artillery unit is hit by the opposing side.

When AU1 performs the combat task, it is assumed that artillery unit AU2 does not move and can be hit [1].

The initial combat capability of artillery unit AU1 is denoted as K_{init}. The mathematical model is built to determine the current combat capability of AU1. It accounts for a decrease due to enemy fire and due to movement between firing positions while continuing the target engagement task [1, 18, 19].

To account for system dynamics, artillery units AU1 and AU2 are considered within three generalized action processes, denoted as A, B, and C.

For the artillery unit AU1, process A corresponds to being at a firing position and includes the following stages:

A1 – transition from march mode to combat mode;

A2 – combat operation against the target;

A3 – transition from combat mode at the firing position to march mode.

Process B corresponds to changing the firing position by AU1.

For AU2, process C corresponds to being at a firing position and includes:

C1 – combat operation against stationary target AU1 from the moment of its first shot until the end of the transition from combat mode to march mode;

C2 – combat operation against moving target AU1 during its firing position change.

The events that occur during mission execution are characterized by the following time intervals:

A1 – time interval of AU1 transition from march mode to combat mode at a firing position: t_{mb};

A2 – time interval of AU1 operation against the target at a firing position for one shot: t_{AU1}, for several shots, the total time increases in proportion to their number;

A3 – time interval of AU1 transition from combat mode at a firing position to march mode: t_{bm};

B – march time when changing the firing position along road j ($i=1,...,s$): t_m^j;

C1 – time interval of AU2 operation against AU1 for one shot: t_{AU2}, and the projectile flight time to a stationary target: t_{st};

C2 – the total interval t_m^j along road j and the interval of AU1 transition from march mode to combat mode at a firing position: t_{bm}.

The current combat capability state of AU1 is influenced by all events that reduce it.

The impact of event A2 (combat operation of AU1 at a firing position) is characterized by:

– a decrease in combat capability due to barrel wear per one shot: k_i^{bar};

– a decrease in combat capability due to chassis wear per one shot: $k_i^{chassis}$.

The impact of event B on the change of AU1 combat capability during transportation along road j is characterized by a decrease due to barrel wear k_{bar}^j and chassis wear $k_{chassis}^j$.

The impact of event C1 on the change of AU1 combat capability is characterized by the decrease value k_{hit}. It is determined by the number of shells d fired from AU2 at AU1 to stop its firing

$$k_{hit} = \sum_{j=1}^{d} \frac{1}{j(j+1)}. \tag{6.19}$$

The number of shells d is computed under the condition that AU1 fires a_i shots at firing position i ($i=1,...,n$)

$$d = \mathrm{INT}\left(\frac{a_i \cdot t_{AU1} + t_{bm} - (t_{AU1} + t_{st})}{t_{AU2} + t_{st}} \right), \tag{6.20}$$

where INT( ) denotes taking the integer part of the resulting value.

The impact of event C2 is characterized by the decrease value k_{move}^j. This decrease is due to unit wear under enemy fire during transportation along road j.

The combat capability of AU1 after combat operation at firing position i is computed as

$$PA_i = PB_{i-1} - (k_{hit} + k_{bar} \cdot a_i + k_{chassis} \cdot t_{AU1} \cdot a_i), \tag{6.21}$$

where PB_{i-1} – the combat capability of AU1 after changing position $i-1 \to i$. By definition, $PB_0 = K_{init}$.

The decrease in combat capability when changing position $i \to i+1$ $(i=1,...,n-1)$ is computed as

$$PB_i = PA_i - (k_{bar}^j + k_{chassis}^j + k_{move}^j). \tag{6.22}$$

The combat capability of AU1 after completing the combat task with *n* shots, if the last shot is performed at position *k* ($k \le n$), equals PA_i.

6.6 Method for finding a solution to the combat employment problem for an artillery unit of the attacking side

For developing a computational method for solving the combat employment problem for the attacking artillery unit under enemy fire, a set of input data is defined for further simulation. Enemy fire affects the current firing position and also affects the unit during position change.

Let's introduce an array $a[1...n]$. Element $a(i)$ defines the number of shots fired at firing position *i*.

Let's also use an array $b[1...n-1]$. Element $b(i)$ corresponds to the number of the road chosen for movement from firing position *i* to the next one.

The current mission structure is an ordered sequence of the numbers of shots at each firing position and the road numbers used for changing positions

$$a(1)b(1)\cdot a(2)b(2)...a(n).$$

Let's set the initial combat capability of AU1 to $K_{init} = 0.965$.

Table 6.2 shows the time intervals of the corresponding actions of AU1 at a firing position, taking into account opposition by the opposing side.

Table 6.3 presents the parameters that characterize how movement between firing positions affects the combat capability of the unit, depending on the chosen road.

Table 6.2 Time intervals of actions during AU1 mission execution at a firing position under opposing fire

Time for	Value
AU1 transition from march mode to combat mode at a firing position, t_{mb}	5 min
AU1 operation against the target at a firing position per one shot, t_{AU1}	15 s
AU2 operation at its firing position against AU1 per one shot, t_{AU2}	20 s
AU1 transition from combat mode at a firing position to march mode, t_{bm}	2 min
Projectile flight time to the target. It defines the start of opposing fire for a stationary target, t_{st}	35 s
Range to the target	12 000 m

Table 6.3 Parameters that affect combat capability when changing positions

Parameters affecting the unit combat capability and the operating time depending on the chosen road	Road No. 1	Road No. 2	Road No. 3
March time when changing the firing position, t_m^j, s	180	720	1440
Decrease in AU1 combat capability due to barrel wear during transportation, k_{bar}^j	0.000025	0.000055	0.000075
Decrease in AU1 combat capability due to chassis wear during transportation, $k_{chassis}^j$	0.00074	0.00094	0.0024
Decrease in AU1 combat capability due to enemy fire impact during transportation, k_{move}^j	0.000055	0.00003	0.000015

The proposed method for computing the combat employment model includes a general algorithm and specialized algorithms:

– the "Positions" algorithm determines the current number of shells used at each position;

– the "Position change" algorithm determines the sequence of road numbers used for movement between positions;

– the "Combat readiness" algorithm determines the final combat capability of AU1 for the current structure;

– the "Time" algorithm determines the total mission time for the current structure.

Using the input data from **Tables 6.2**, **6.3**, the general algorithm forms the current mission structure $a(1)b(1)\cdot a(2)b(2)...a(n)$. It then computes the final combat capability of the artillery unit accounting for losses due to enemy fire, as well as the movement between firing positions to continue the target engagement task.

Before presenting a formal algorithm description, it is useful to explain the logic of forming the mission structure using a simplified example.

Assume that artillery unit AU1 performs a task from two firing positions. Movement between the first and the second position is possible along several alternative routes. They differ in march duration and in their impact on combat capability. At each position, AU1 can fire one or several shots and then change position.

In this case, the mission structure is an ordered sequence of decisions which includes the number of shots fired at each firing position and the choice of the movement route between neighboring positions.

For two firing positions, such a structure can be written as

$$a(1)b(1)a(2).$$

Here $a(1)$ and $a(2)$ define the number of shots at the first and second positions, respectively. Value $b(1)$ is the road number chosen for movement between them.

For example, the structure $a(1)=2$, $b(1)=3$, $a(2)=1$ means two shots at the first position, movement to the second position along Road No. 3, and one shot at the second position.

For a given structure, the total mission time is the sum of deployment time at each position, firing time, packing time, and the march time between positions. For each firing position, let's use the time intervals in **Table 6.2**. For movement, let's use the march time t_m^j, from **Table 6.3**. Thus, the chosen road $b(1)$ directly affects the total mission time.

Similarly, the final combat capability is computed by accounting for its decrease during firing and during movement between positions. During position change, it is possible to account for barrel wear, chassis wear, and enemy impact during movement. These decreases depend on the chosen route and are given by k_{bar}^j, $k_{chassis}^j$ and k_{move}^j (**Table 6.3**).

It is evident that another admissible structure, for example, $a(1)=1$, $b(1)=1$, $a(2)=2$, leads to different values of total time and to a different level of combat capability decrease. Fewer shots at the first position reduce the time under enemy fire. A shorter route reduces march time and changes the losses during transportation.

For an arbitrary number of firing positions, the mission structure is a sequence of decisions

$$a(1)b(1)a(2)b(2)...a(n).$$

This sequence uniquely defines a combat scenario for the artillery unit. A comparative analysis of all admissible structures makes it possible to find those that provide extreme values of mission time and of preservation combat capability. This is implemented by the algorithm below.

For clarity, let's recall the meaning of the indices used below:

– i – the firing position number where combat operation is performed;

– j – the road number chosen for movement between firing positions;

– n – the total number of firing positions in the mission;

– k – the number of the last firing position in the current mission structure.

All indices below are used in accordance with the given explanations.

The general algorithm is implemented as the following sequence of steps:

Step 1. Set the initial mission scenario by taking $a(1)=\cdots a(n)=1$. Thus, one shot is planned at each firing position. Here $k=n$ is the number of positions in the scenario.

Step 2. Initialize variables for extreme values of combat capability and mission time: $P_{max} = 0$ (maximum combat capability), $P_{min} = 1$ (minimum combat capability), $T_{max} = 0$ (maximum total time), and $T_{min} = 10^{10}$ (minimum total time).

Step 3. Set the initial movement scenario by taking $b(1) = \cdots b(k-1) = 1$. Thus, Road No. 1 is chosen at each movement stage.

Step 4. For the current scenario, compute the final combat capability of AU1 using the "Combat capability" algorithm.

Step 5. For the current scenario, compute the total mission time using the "Time" algorithm. The time includes firing-position time and movement time.

Step 6. Compare the obtained final combat capability $PA(k)$ and the total time T with the current extreme values P_{max}, P_{min}, T_{max}, and T_{min}.

If, for the current scenario, combat capability or total time values exceed or, respectively, decrease registered before extreme values, update them. Store the mission structure arrays $a[1...n]$ and $b[1...n-1]$ that correspond to the new extreme values of combat capability or time.

Step 7. For a fixed number of shots across firing positions, enumerate all admissible scenarios of movement between positions. For this, use the "Position change" algorithm to form the next variant of array $b[1...k-1]$ corresponding to another choice of routes between positions.

After forming a new movement scenario, repeat Steps 4–6 to compute the final combat capability and the total time of the mission. Continue until all route combinations for the current shot structure are considered.

Step 8. After enumerating all admissible movement scenarios for the current shot structure, form the next scenario of distributing shots across firing positions. Use the "Positions" algorithm to change elements of array $a[1...n]$.

For a new shot structure, re-initialize enumeration of movement scenarios by setting $b(1) = \cdots b(k-1) = 1$, and return to Step 3. Continue until all admissible shot distributions are considered.

Step 9. After enumerating all admissible mission structures, determine the final extreme values of combat capability and mission time, as well as corresponding scenarios.

Computation results include final values P_{max}, P_{min}, T_{max}, and T_{min}, and mission structures – arrays $a[1...n]$ and $b[1...n-1]$ which provide these extreme values.

Step 10. Form the algorithm outputs. They include the numerical values of combat capability and mission time, and a description of the corresponding mission scenarios.

Step 11. Output the results in a form convenient for further analysis. The algorithm is complete.

The "Positions" algorithm takes the input array $a[1...k]$, which defines the current mission structure within the general algorithm:

Step 1. Determine *m*, the index of the last non-zero element of array *a* of the current mission structure.

Step 2. Increase the number of shots at the previous firing position

$$a(m-1)=a(m-1)+1.$$

Step 3. Adjust element $a(m)$ with the constraint on the total number of shots:

– if $a(m)=1$, set $a(m)=0$;

– if $a(m)=n-m$, then sequentially for $a(m+1),...,a(m+i)$ set the value 1 while the constraint $\sum_{j=1}^{m+i} a(j)\le n$ is satisfied;

– otherwise, decrease $a(m)$ by 1: $a(m)=a(m)-1$.

Step 4. Return the updated array $a[1...k]$ which defines the next admissible mission structure.

The "Position change" algorithm takes the input array $b[1...k-1]$, which defines the chosen roads for movement between consecutive firing positions within the current mission structure:

Step 1. Determine *m*, the index of the last element of array *b*, which value equals 1 or 2.

Step 2. Form the next structure of position change:

– if $m=k-1$, the value of the last element increases: $b(k-1)=b(k-1)+1$;

– otherwise, increase $b(m)$ by 1 and set all following elements to 1:

$$b(m)=b(m)+1,$$

$$b(i)=1, (i=m+1...k-1).$$

Step 3. Return the updated array $b[1...k-1]$. It defines the next admissible variant of choosing a route between firing positions.

The "Combat capability" algorithm takes the input arrays $a[1...k]$ and $b[1...k-1]$ from the general algorithm. They define the current mission structure for artillery unit AU1:

Step 1. Set the initial combat capability of AU1 at the first firing position as $PB(1)=K_{init}$.

Step 2. For each firing position $i=1...k$, perform Steps 3–7.

Step 3. Compute the number of shells *d* according to equation (6.20).

Step 4. Compute the combat capability decrease coefficient of AU1 due to hits from AU2 according to equation (6.19).

Step 5. Compute the combat capability decrease of AU1 during firing against the target

$$k_{wear} = (k_{bar} + k_{chassis}) \cdot t_{\text{AU1}} \cdot a(i). \tag{6.23}$$

Step 6. Compute the combat capability of AU1 after work at position i

$$PA_i = PB_i - (k_{hit} + k_{wear}). \tag{6.24}$$

Step 7. For movement between neighboring firing positions i and $i+1$ ($i = 1...k-1$), compute the decrease of AU1 combat capability due to movement along the selected route

$$PB_{i+1} = PA_i - (k_{bar}^{b(i)} + k_{chassis}^{b(i)} + k_{move}^{b(i)}). \tag{6.25}$$

Step 8. The algorithm output is $PA(k)$, the combat capability of AU1 corresponding to the current mission structure.

The "Time" algorithm takes the input $a[1...k]$ and $b[1...k-1]$ from the general algorithm. They define the current mission structure for artillery unit AU1:

Step 1. Set the initial total mission time as $T = 0$.

Step 2. For each firing position $i = 1...k$, perform Steps 3–4.

Step 3. Add the duration of AU1 operation at the i-th firing position, including deployment, firing, and packing time

$$T = T + t_{mb} + a(i) \cdot t_{\text{AU1}} + t_{bm}. \tag{6.26}$$

Step 4. If $i \neq k$, add the march time to the next firing position. It is determined by road choice $b(i)$

$$T = T + t_{bm}^{b(i)}. \tag{6.27}$$

Step 5. The algorithm output is the total time T for AU1 mission execution for the current mission structure.

6.7 Conclusions

The study of the decrease in combat capability of an artillery unit, in the problem setting of Section 6.3, corresponds to selecting a mission structure from a large but finite set of possible options. Formally, such a problem can be treated as Pareto-oriented or as a dynamic programming problem.

However, the limited number of input parameters and variable arguments made it possible to obtain all possible solutions by a full direct enumerating. The simulation results for the selected modeling options are given in **Tables 6.6**, **6.7**.

The characteristics of variable arguments used in the simulation experiment are presented in **Table 6.4**. They define admissible values for the number of shots at firing positions and parameters that influence the mission structure.

Table 6.4 Characteristics of variable arguments of the combat employment model

Characteristic	Value 1	Value 2	Value 3	Value 4
1. Start of opposing fire after the first shot of the attacker t_{st}, s	35	43	51	59
2. Number of shells to destroy a stationary target *n*, pcs	4	6	8	10

Parameters that characterize the change in combat capability during movement between firing positions, depending on the selected route, are given in **Table 6.5**. These parameters account for wear of the main system components and the influence of the opposing side during the march.

Table 6.5 Parameters that affect combat capability when changing a firing position

Combat capability reduction parameter for AU1	Modeling option *X*			Modeling option *Y*		
	Road					
	1	2	3	1	2	3
1. Due to barrel wear during transportation, k^j_{bar}	$2.5 \cdot 10^{-5}$	$5.5 \cdot 10^{-5}$	$7.5 \cdot 10^{-5}$	$2.5 \cdot 10^{-4}$	$5.5 \cdot 10^{-4}$	$7.5 \cdot 10^{-4}$
2. Due to chassis wear during transportation, $k^j_{chassis}$	$7.4 \cdot 10^{-4}$	$9.4 \cdot 10^{-4}$	$2.4 \cdot 10^{-3}$	$7.4 \cdot 10^{-3}$	$9.4 \cdot 10^{-3}$	$2.4 \cdot 10^{-2}$
3. Due to wear under enemy fire during transportation, k^j_{move}	$5.5 \cdot 10^{-5}$	$3.0 \cdot 10^{-5}$	$1.5 \cdot 10^{-5}$	$5.5 \cdot 10^{-4}$	$3.0 \cdot 10^{-4}$	$1.5 \cdot 10^{-4}$

Table 6.6 Distribution of all possible combat capability values for 10 shots for two modeling options

Distribution interval	Modeling option *X*				Modeling option *Y*			
	Fire start, t_{st}, s				Fire start, t_{st}, s			
	35	43	51	59	35	43	51	59
1	2	3	4	5	6	7	8	9
(0.075; 0.100]	0	0	0	0	37	0	0	0
(0.100; 0.125]	0	0	0	0	120	2	0	0
(0.125; 0.150]	202	7	1	0	53	5	1	0
(0.150; 0.175]	0	0	0	0	833	0	0	0
(0.175; 0.200]	0	0	0	0	6144	6	0	0
(0.200; 0.225]	0	0	0	0	23026	325	5	0
(0.225; 0.250]	0	0	0	0	35117	1350	176	4
(0.250; 0.275]	0	0	0	0	16902	2217	471	81
(0.275; 0.300]	120	0	0	0	1373	941	355	110
(0.300; 0.325]	83283	4851	1023	208	0	12	16	13
(0.575; 0.600]	0	0	0	0	1	1	1	1
(0.600; 0.625]	0	0	0	0	171	171	171	171
(0.625; 0.650]	0	0	0	0	1978	1986	1986	1986
(0.650; 0.675]	0	0	0	0	13363	14196	14196	14196
(0.675; 0.700]	0	0	0	0	44613	50766	50772	50772
(0.700; 0.725]	0	0	0	0	65746	89247	89567	89572
(0.725; 0.750]	0	0	0	0	37652	74350	75524	75696
(0.750; 0.775]	0	0	0	0	5670	24651	26397	26787
(0.775; 0.800]	8086	8206	8206	8206	15	1915	504	2749
(0.800; 0.825]	161123	249077	252914	253730	0	0	2	6
Structures and corresponding extreme values of combat capability								
For max combat capability value	**Shot number sequence**							
	2, 2, 2, 2, 2	2, 4, 4	5, 5	4, 6	2, 2, 2, 2, 2	2, 4, 4	5, 5	4, 6
	Road number sequence while changing positions							
	1, 1, 1, 1	1, 1	1	1	1, 1, 1, 1	1, 1	1	1
	Combat capability value							
	0.8117	0.8134	0.8142	0.8142	0.7822	0.7986	0.8068	0.8068
	Time							
	2680	1590	1050	1050	2680	1590	1050	1050

Continuation of Table 6.6

1	2	3	4	5	6	7	8	9
For min combat capability value	Shot number sequence							
	1, 1, 1, 7	1, 9	10	1, 1, 1, 7	1, 1, 1, 7	1, 9	10	1, 1, 1, 7
	Road number sequence while changing positions							
	3, 3, 3	3		3, 3, 3	3, 3, 3	3		3, 3, 3
	Combat capability value							
	0.1409	0.1458	0.1483	0.3075	0.0736	0.1234	0.1483	0.2403
	Time							
	5910	2310	510	5910	5910	2310	510	5910

Table 6.7 Distribution of all possible combat capability values for 4 shots for two modeling options

Distribution interval	Modeling option *X*				Modeling option *Y*			
	Fire start, t_{st}, s				Fire start, t_{st}, s			
	35	43	51	59	35	43	51	59
(0.375; 0.400]	0	0	0	0	6	0	0	0
(0.400; 0.425]	7	0	0	0	1	0	0	0
(0.825; 0.850]	0	0	0	0	7	7	7	7
(0.850; 0.875]	0	0	0	0	28	28	28	28
(0.875; 0.900]	7	7	7	7	22	28	28	28
(0.900; 0.925]	50	57	57	57	0	1	1	1
Structures and corresponding extreme values of combat capability								
For max combat capability value	Shot number sequence							
	2,2	4	4	4	2,2	4	4	4
	Road number sequence while changing positions							
	1	–	–	–	1	–	–	–
	Combat capability value							
	0.9042	0.905	0.905	0.905	0.8968	0.905	0.905	0.905
	Time							
	960	420	420	420	960	960	960	960
For min combat capability value	Shot number sequence							
	1, 3	1, 1, 1, 1	1, 1, 1, 1	1, 1, 1, 1	1, 3	1, 1, 1, 1	1, 1, 1, 1	1, 1, 1, 1
	Road number sequence while changing positions							
	3	3, 3, 3	3, 3, 3	3, 3, 3	3	3, 3, 3	3, 3, 3	3, 3, 3
	Combat capability value							
	0.4025	0.8975	0.8975	0.8975	0.3801	0.8303	0.8303	0.8303
	Time							
	2220	5820	5820	5820	2220	5820	5820	5820

During the simulation experiment, it was assumed that all shots are effective [1, 17]. The hit probability for the artillery unit was assumed to be at least 0.5. The decrease in combat capability of the attacking artillery unit due to hits by the opposing side between consecutive shots is described by the relation used in Step 4 of the "Combat capability" algorithm.

The results in **Tables 6.6**, **6.7** characterize mission execution for a large series of shots (10 shots) and for a small series of shots (4 shots), respectively. For each series, the best and the worst options are shown in terms of combat capability loss and mission time.

It is noteworthy that the computed combat capability value can be negative in some cases. From a physical point of view, this corresponds to the loss of the artillery unit. The larger the absolute value of this negative number, the earlier the loss occurs at previous stages of the mission for the considered structure.

The analysis in **Tables 6.6**, **6.7** shows that for each considered shot series there exists a mission structure that provides the minimum decrease in combat capability at an admissible mission time.

At the same time, optimizing the structure only for maximum combat capability preservation does not always lead to the minimum mission time. Conversely, the structures that provide the minimum time can, in some cases, cause a significant decrease in combat capability, up to the physical loss of the artillery unit due to opposing fire.

For the considered case, 262,144 combat capability values are computed for each modeling option, taking into account changes in all computational arguments.

This makes it possible to quantitatively assess the dispersion of admissible combat scenarios and to identify mission structures that ensure a 4–10 times higher preservation of combat capability compared to the worst-case solutions.

For a 10-shot mission in modeling option *X*, the initial combat capability of 0.965 changed within a wide range. The highest concentration of results (169,209 values) corresponds to the start of opposing fire at 35 s. In this case, combat capability varies from 0.775 to 0.825. Similar bands were observed for other moments of time: 43 s gave 257,283 values, 51 s gave 261,120 values, and 59 s gave 261,936 values.

In modeling option *Y*, combat capability also decreased from 0.965. However, the band for 35 s is wider, from 0.575 to 0.825, with 169,209 results. For other moments of time, the value bands are similar: 43 s gives 257,283 values, 51 s gives 261,120 values, and 59 s gives 261,936 values.

In both models (*X* and *Y*), 9.330 negative combat capability values were observed at 35 s. This indicates the actual loss of the artillery unit. For other moments of time, negative values did not occur.

The analysis of shot allocation showed that optimal solutions range from a uniform distribution, with two shots at each of the first five firing positions, to a concentrated fire pattern with five shots from each of the first two positions.

When choosing movement routes between positions, the best results correspond to the faster but more dangerous road option (Road No. 1).

The ratio of combat capability values for the best and the worst solutions in the considered cases ranges from 4 to 10. From a practical point of view, this means that an informed choice of shot distribution and movement routes allows either to preserve combat capability above 0.8 or, alternatively, to significantly reduce mission time at the cost of controlled combat capability degradation.

Qualitatively similar patterns are observed analyzing the results of **Tables 6.6**, **6.7**.

When the number of shots decreases, the total number of admissible mission structures reduces. The range of final combat capability values becomes narrower. The ratio between the best and the worst options becomes less than 2.

At the same time, for a smaller number of shots, the number of possible firing positions increases. This raises the variability of the mission structure.

The analysis of mission time based on **Tables 6.6**, **6.7** shows that for most computed structures, the best option in terms of combat capability preservation also corresponds to the minimum mission time.

However, for scenarios with a large number of shots, solutions exist where mission time decreases significantly. This is achieved by a sharp reduction of combat capability to critical values. Combat capability values in the range 0.1–0.2 correspond, in practical terms, to the actual loss of the artillery unit due to opposing fire.

The analysis of **Table 6.7** shows that the artillery unit can be employed with no more than two shots at each firing position without a significant decrease in combat capability. Under these conditions, an acceptable level of combat survivability is maintained throughout the mission.

If a mission with up to ten shots is oriented mainly to defensive conditions, then engaging a target with no more than four shots from one firing position is more typical for offensive operations. In this case, reducing the time spent under opposing fire plays a key role [1, 8].

This is confirmed by the worst scenarios considered. In these scenarios, movement between firing positions is performed without accounting for route characteristics and the related combat capability losses. Such scenarios correspond to intensive offensive operations. In them, priority is given to reducing mission time, even at the cost of a large decrease in combat capability.

In this context, the classical "shoot and scoot" tactic [20, 21], which is typical for offensive operations, can be interpreted, based on the modeling results, as a "hide and shoot" tactic. This interpretation prioritizes concealment and minimization of the time under opposing fire.

The simulation experiment shows that the developed simulation model and the state-control method for the artillery unit were further developed by including random dynamic external and internal disturbances that accompany mission execution.

As a result, the proposed method provides both quantitative advantages, expressed in numerical estimates of combat capability, mission time and their ratios, and qualitative advantages, such as the ability to formally compare offensive and defensive employment strategies within a unified modeling framework.

Conflict of interest

The authors declare that they have no conflict of interest in relation to this research, whether financial, personal, authorship or otherwise, that could affect the research and its results presented in this paper.

Use of artificial intelligence statement

The authors declare that they did not use artificial intelligence tools in preparing this manuscript.

Authors' contributions

Pavlo Gultsov: Conceptualization, Methodology, Development of stochastic and simulation models, Writing – original draft.

Oksana Maksymova: Data analysis, Numerical simulations, Validation of models, Visualization of results.

Yevhenii Dobrynin: Formal analysis, Interpretation of simulation outcomes, Writing – review & editing.

References

1. Boltenkov, V., Brunetkin, O., Dobrynin, Y., Maksymova, O., Kuzmenko, V., Gultsov, P. et al. (2021). Devising a method for improving the efficiency of

artillery shooting based on the Markov model. Eastern-European Journal of Enterprise Technologies, 6 (3 (114)), 6–17. https://doi.org/10.15587/1729-4061.2021.245854

2. Dobrynin, Y., Brunetkin, O., Maksymov, M., Maksymov, O. (2020). Constructing a method for solving the riccati equations to describe objects parameters in an analytical form. Eastern-European Journal of Enterprise Technologies, 3 (4 (105)), 20–26. https://doi.org/10.15587/1729-4061.2020.205107
3. Brunetkin, O., Beglov, K., Brunetkin, V., Maksymov, O., Maksymova, O., Havaliukh, O., Demydenko, V. (2020). Construction of a method for representing an approximation model of an object as a set of linear differential models. Eastern-European Journal of Enterprise Technologies, 6 (2 (108)), 66–73. https://doi.org/10.15587/1729-4061.2020.220326
4. Brunetkin, O., Maksymov, M., Brunetkin, V., Maksymov, O., Dobrynin, Y., Kuzmenko, V., Gultsov, P. (2021). Development of the model and the method for determining the influence of the temperature of gunpowder gases in the gun barrel for explaining visualize of free carbon at shot. Eastern-European Journal of Enterprise Technologies, 4 (1 (112)), 41–53. https://doi.org/10.15587/1729-4061.2021.239150
5. Maksymova, O., Boltyonkov, V., Gultsov, P., Maksymov, O. (2023). Improvement of the model and method of artillery installation target damage control with minimal combat capability loss. Proceedings of Odesa Polytechnic University, 2 (68), 98–115. https://doi.org/10.15276/opu.2.68.2023.11
6. Tarakhtii, O. S., Gultsov, P. S., Maksymov, O. M. (2023). Pat. No. 127193. Sposib vyznachennia koordynaty zustrichi artyleriiskoho snariada z poverkhneiu. Declareted: 28.04.2021; published: 31.05.2023, Bul. No. 22.
7. Tarakhtii, O. S., Gultsov, P. S., Maksymov, O. M. (2024). Udoskonalennia modeli keruvannia boiovoiu zdatnistiu artyleriiskoi harmaty. Topical Aspects of Modern Scientific Research. Proceedings of the 5th International Scientific and Practical Conference. Tokyo: CPN Publishing Group, 256–261. Available at: https://sci-conf.com.ua/v-mizhnarodna-naukovo-praktichna-konferentsiya-topical-aspects-of-modern-scientific-research-25-27-01-2024-tokio-yaponiya-arhiv/
8. Tarakhtii, O. S., Gultsov, P. S., Maksymov, O. M. (2024). Metod parabolichnoi aproksymatsii vyznachennia koordynaty zitknennia artyleriiskoho snariada z poverkhneiu. Modern Problems of Science, Education and Society. Proceedings of the 12th International Scientific and Practical Conference. Kyiv, 324–330. Available at: https://sci-conf.com.ua/xii-mizhnarodna-naukovo-praktichna-konferentsiya-modern-problems-of-science-education-and-society-5-7-02-2024-kiyiv-ukrayina-arhiv/

9. Maksymov, M. V., Boltenkov, V. O., Gultsov, P. S., Maksymov, O. M. (2023). Verification of artillery fire under the influence of random disturbances for the computer game ARMA 3. Applied Aspects of Information Technology, 6 (4), 362–375. https://doi.org/10.15276/aait.06.2023.24
10. Dobrynin, Y. V., Boltenkov, V. O., Maksymov, M. V. (2020). Information technology for automated assessment of the artillery barrels wear based on SVM classifier. Applied Aspects of Information Technology, 3 (3), 117–132. https://doi.org/10.15276/aait.03.2020.1
11. Dobrynin, Y., Maksymov, M., Boltenkov, V. (2020). Development of a method for determining the wear of artillery barrels by acoustic fields of shots. Eastern-European Journal of Enterprise Technologies, 3 (5 (105)), 6–18. https://doi.org/10.15587/1729-4061.2020.206114
12. Maksimova, O. B., Davydov, V. O., Babych, S. V. (2016). Optimization of Control of Heat Supply Systems of Urban Districts. Journal of Automation and Information Sciences, 48 (4), 69–89. https://doi.org/10.1615/jautomatinfscien.v48.i4.70
13. Dobrynin, Y., Volkov, V., Maksymov, M., Boltenkov, V. (2020). Development of physical models for the formation of acoustic waves at artillery shots and study of the possibility of separate registration of waves of various types. Eastern-European Journal of Enterprise Technologies, 4 (5 (106)), 6–15. https://doi.org/10.15587/1729-4061.2020.209847
14. Maksymov, M. V., Brunetkin, O. I., Beglov, K. V., Alyokhina, S. V., Butenko, O. V. (2022). Automatic Control for the Slow Pyrolysis of Organic Materials with Variable Composition. Advanced Control Systemss: Theory and Applications. Series in Automation, Control and Robotics. River Publishers, 397–434. https://doi.org/10.1201/9781003337010-16
15. Brunetkin, O., Maksymov, M., Dobrynin, Y., Demydenko, V., Sidelnykov, O. (2024). Development of a process model for determining the composition and energy characteristics of a pyrotechnic mixture using the library method. EUREKA: Physics and Engineering, 5, 99–112. https://doi.org/10.21303/2461-4262.2024.003453
16. Brunetkin, O., Dobrynin, Y., Maksymenko, A., Maksymova, O., Alyokhina, S. (2020). Inverse problem of the composition determination of combustion products for gaseous hydrocarbon fuel. Computational Thermal Sciences: An International Journal, 12 (6), 477–489. https://doi.org/10.1615/computthermalscien.2020034878
17. Brunetkin, O. I., Beglov, K. V., Maksymov, M. M., Ulytska, O. O. (2021). Model and method of controlled pyrolysis of organic sub-stances of variable

composition. Problems of Control and Informatics, 66 (1), 134–146. https://doi.org/10.34229/1028-0979-2021-1-12
18. Brunetkin, O., Dobrynin, Y., Maksymenko, A., Maksymova, O., Alyokhina, S. (2020). Model and method of conditional formula determination of oxygen-containing hydrocarbon fuel in combustion. Energetika, 66 (1). https://doi.org/10.6001/energetika.v66i1.4298
19. Brunetkin, O., Sidelnykov, O., Maksymov, M., Dobrynin, Y. (2025). Improving the model for determining the composition of gunpowder gases during thermal destruction of gunpowder in a limited volume space. Eastern-European Journal of Enterprise Technologies, 3 (6 (135)), 35–45. https://doi.org/10.15587/1729-4061.2025.330654
20. Shim, Y., Atkinson, M. P. (2018). Analysis of artillery shoot-and-scoot tactics. Naval Research Logistics, 65 (3), 242–274. https://doi.org/10.1002/nav.21803
21. Koba, M. (1996). Artillery Strike Force. Fort Leavenworth: School of Advanced Military Studies, United States Army Command and General Staff College.