CHAPTER 8

Method of parabolic approximation for determining the impact point coordinates of an artillery projectile

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Abstract

This section considers a method for verifying an artillery shot under conditions of random disturbances, which is based on the registration of acoustic fields formed by ballistic and muzzle waves. The position of the proposed approach among modern technologies for ensuring the accuracy of artillery fire is demonstrated, and the feasibility of using the ballistic wave as a source of useful information for estimating projectile flight parameters is substantiated.

A general description of the method, the layout of the measuring equipment, and the sequence of measurement data processing are presented. The method is based on recording the moments when the projectile passes over spatially separated observation points, followed by approximation of the flight trajectory using a system of parabolas. The proposed algorithm takes into account possible changes in the relative positions of the measurement points with respect to the ascending and descending branches of the trajectory, which makes it possible to partially compensate for random disturbances caused by instability of the initial velocity and other firing-related factors.

The effectiveness of the method is investigated by means of simulation modeling for a large-caliber artillery projectile, taking into account random disturbances of temporal parameters. It is shown that the use of a system of approximating parabolas provides estimation of the projectile impact point coordinates with an error on the order of fractions of a percent of the firing range. A comparative analysis with the traditional method of compensating random disturbances by successive corrective shots is carried out, and the results of a field experiment involving the registration of ballistic wave signals are presented. The obtained results confirm

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the fundamental possibility of verifying an artillery shot using a single firing with acceptable accuracy.

Keywords

Artillery shot, firing verification, ballistic wave, acoustic reconnaissance, trajectory approximation, random disturbances.

8.1 Introduction

Modern conditions of employment of artillery units are characterized by high dynamics of combat operations, an increasing role of counter-battery warfare, and strict limitations on the duration of fire impact. Under such conditions, the effectiveness of artillery fire is determined not only by the accuracy of ballistic calculations but also by the ability to promptly verify the results of a shot and rapidly adjust fire taking into account real firing conditions. This, in turn, necessitates a transition from classical counter-battery engagement schemes to short-duration fire tactics with immediate relocation of firing positions of the "shoot-and-scoot" type [1, 2].

Achieving the required level of combat effectiveness of artillery units under such conditions is impossible without extensive use of modern information technologies, automated fire control systems, and specialized geoinformation tools. The calculation of artillery projectile flight trajectories, the construction of firing tables, and the assessment of target engagement accuracy are based on refined mathematical models, methods of optimal estimation, and forecasting. The universality of this mathematical framework is confirmed by its application in modeling complex dynamic and physicochemical processes, in particular in parameter identification problems, the solution of inverse problems, and the development of approximation models for systems of various physical nature [3–10].

A special place among the means of verifying artillery firing results is occupied by acoustic methods based on the registration and processing of sound signals generated by the shot and the projectile burst. Despite the complex nature of acoustic wave propagation under real conditions, modern signal processing and mathematical modeling techniques make it possible to obtain informative temporal and spatial parameters suitable for estimating burst coordinates and analyzing firing errors [1, 2].

In view of the above, a relevant scientific and applied task is the development and investigation of methods for determining the coordinates of the artillery projectile impact point based on acoustic measurements using simplified yet informative trajectory approximation models. The use of parabolic approximation makes it possible to reduce the computational complexity of the problem, ensure sufficient estimation accuracy with a limited amount of measurement information, and create prerequisites for the practical implementation of firing result verification methods in a mode close to real time [3, 4].

At the same time, the proposed approach is based on a number of assumptions that define the scope of its applicability. It is assumed that the projectile motion remains supersonic over the observation segment and that the ballistic wave can be reliably registered at spatially distributed measurement points. The method relies on the availability of sufficiently accurate firing tables for estimating the initial projectile velocity and does not explicitly model complex aerodynamic effects beyond integral drag. In addition, the proposed approximation framework is intended for conditions where meteorological parameters vary slowly in space and time and can be treated as quasi-stationary during a single shot.

8.2 General principles of acoustic reconnaissance tasks

A method for determining the coordinate of the artillery projectile impact with the surface by means of acoustic registration of the acoustic field generated by the projectile is considered. The registration of the acoustic field of a projectile that detonates near the target can be regarded as an integral process, the result of which is a reduction in projectile expenditure during mission execution by avoiding computational and measurement errors, as well as by reducing projectile dispersion under compensating actions of external disturbances recorded over a series of shots. The rate of introducing compensating actions into the operation of artillery units is a decisive factor in the conduct of combat operations under modern conditions.

Acoustic reconnaissance is conducted continuously, and the obtained data must be reliable and accurate. Based on the coordinates of acoustic targets, fire for effect may be conducted without prior adjustment. At the same time, based on the coordinates of projectile impact and burst points, adjustment fire should be carried out for non-acoustic and optically undetected targets.

The registration of the acoustic field for the needs of artillery units makes it possible to solve two main tasks. For this purpose, the equipment of a sound-ranging complex is deployed in a specific configuration in the field, consisting of several sound receivers placed at base points, as well as a central point that accommodates information processing equipment, an observation post, and communication facilities [11].

The first task makes it possible to detect firing positions of guns that are not optically observable, regardless of the types of ammunition used, based on a demasking feature, namely the sound of the shot. Below, a generalized sequence of operations performed during the deployment of acoustic reconnaissance units to ensure fire support of artillery guns against acoustic targets is presented:

Step 1. The acoustic reconnaissance unit is deployed on the basis of an automated sound-ranging complex that includes three sound receivers placed at base points with known coordinates. At the central point, the coordinates of which are also known, information processing equipment is installed, consisting of a central computing unit, a meteorological measurement complex, and observation and communication facilities. The sound receivers and the information processing equipment are connected via internal communication means, while interaction between the artillery unit and the sound-ranging complex is ensured by external communication means.

Step 2. The sound receivers are deployed in the field under identical conditions, preferably on dominant elevations, along a line perpendicular to the expected direction of acoustic wave propagation, at a distance of 500–1000 m from each other.

Step 3. The coordinates of each sound receiver are determined and entered into the information processing equipment of the central computing unit.

Step 4. The meteorological measurement complex records meteorological information (air temperature, wind direction, and wind speed in the near-surface atmospheric layer), which is transmitted to the central point and entered into the information processing equipment of the central computing unit.

Step 5. Adjustment and calibration of the information reception equipment are performed at each base point with connected sound receivers.

Step 6. Separate adjustment of the interaction channels between the equipment located at the base points and the information processing equipment of the central point is carried out.

Step 7. Calibration of the information transmission channels from each sound receiver to the information processing equipment is performed.

Step 8. Astronomical time is entered into the central computing unit, and synchronization and alignment of the unified time of the central point with the time at each base point are carried out.

The second task makes it possible to conduct fire by friendly artillery against non-acoustic and optically undetected targets by determining the projectile impact points based on the sound of their bursts. Below, a sequence of operations performed during the support of artillery gun fire by an acoustic reconnaissance unit based on projectile bursts is presented:

Step 1. The coordinates of the target and the firing position are entered into the central computing unit.

Step 2. A shot is fired from the artillery gun at the firing position. At the moment the projectile collides with a solid obstacle, it detonates, resulting in the generation of a sound wave which propagates through the atmosphere and reaches the sound receivers.

Step 3. Using the sound receivers, the sound wave generated by the projectile burst is recorded, and the acquired information is transmitted via a communication channel to the information processing equipment. As a result of processing, the central computing unit determines the coordinates of the projectile burst point (reference point).

Step 4. The coordinates of the burst point are calculated based on the known positions of the sound receivers, the sound wave propagation velocity with corrections for the current meteorological conditions of the near-surface atmospheric layer, and the differences in sound wave arrival times at the sound receivers.

Step 5. The coordinates of the projectile impact point relative to the firing position are determined.

Artillery gun fire conducted with the support of an acoustic reconnaissance unit according to the above algorithms during the execution of combat tasks has a number of limitations caused by the physical properties of the medium of acoustic wave propagation, the nature of the surface in the projectile burst zone, and the presence of natural or artificial obstacles along the sound signal propagation path [12].

The above factors should be regarded as inherent limitations of acoustic-based methods and define the conditions under which the proposed approach can be effectively applied.

When firing at targets located on soft marshy soil or on ground covered with a thick snow layer, a complete projectile detonation may not occur upon impact with such a surface. Under these conditions, an acoustic wave capable of propagating in the atmosphere and reaching the sound receivers is not formed, which makes it impossible to register the burst moment and, accordingly, to determine the coordinates of the projectile impact point.

When firing at targets located on a water surface, the projectile passes through the water layer and detonates in the underwater environment. In this case, the main part of the explosion energy is absorbed by the water, as a result of which an acoustic wave in the atmosphere is either not formed or has an intensity insufficient for reliable registration by the sound receivers.

Significant attenuation of the acoustic wave is also possible even when a projectile burst occurs, if its propagation path passes through media with a high absorption coefficient of acoustic oscillations. Such conditions include large forested areas as well as complex terrain with the presence of hills, ravines, and other natural obstacles, which lead to a reduction in signal amplitude and a deterioration in registration accuracy.

The registration of the acoustic wave generated by a projectile explosion is carried out by sound receivers arranged along a line perpendicular to the direction of the expected arrival of the acoustic wave.

The listed limitations may lead to the impossibility of determining the coordinates of the projectile burst or impact point, either due to the absence of an acoustic signal or under conditions of its significant attenuation during atmospheric propagation. These factors must be taken into account when employing acoustic reconnaissance means to support artillery fire.

8.3 Method for registering the coordinate of artillery projectile impact with the surface

The method for determining the coordinate of an artillery projectile impact with the surface is based on an approach in which, through the introduction of new operations and a modification of the execution order of existing ones, it becomes possible to determine the coordinates of unexploded projectiles and to increase the accuracy of determining the projectile impact coordinate. The following features are relevant for the proposed method [13].

Along the firing direction line from the gun, at a presumed distance at which the projectile loses its supersonic velocity, locations for the placement of three measuring microphones or three groups of measuring microphones are determined.

The conditions for placing three measuring microphones or three groups of measuring microphones are specified:

1) at predetermined locations, the first, second, and third measuring microphones, or the first, second, and third groups of measuring microphones, are deployed with a positioning error not exceeding 50% of the specified distances;

2) the coordinates of the three measuring microphones or three groups of measuring microphones located along the firing line are determined;

3) the ballistic and muzzle waves are recorded above the microphones or groups of microphones;

4) the time interval between the registration of the ballistic wave and the registration of the muzzle wave is determined for each measuring microphone or

group of measuring microphones, from the microphone or group closest to the gun to the microphone or group farthest from the gun;

5) the initial projectile velocity at the moment of firing is determined;

6) three types of ballistic curves defined by three points are obtained, based on which the coordinates of the artillery projectile impact with the surface are determined;

7) using the three obtained coordinates, the average coordinate of the artillery projectile impact with the surface is determined.

The causal relationship between the set of presented conditions and the achieved technical result is explained as follows.

At the moment the projectile exits the barrel (the sound source), two sound waves are generated. The ballistic wave is formed when the projectile moves at supersonic velocity and exists over the time interval until it disappears as a result of the projectile transitioning to a subsonic motion regime, where the propagation velocity of the wave is equivalent to the instantaneous velocity of the projectile. The muzzle wave is formed by the outflow of propellant gases after the projectile leaves the barrel. Initially, over a relatively short time interval, it exhibits the properties of a shock-acoustic wave and transforms into an acoustic wave as the pressure of the gases equalizes with atmospheric pressure [14]. Subsequently, its propagation velocity becomes constant and equal to the speed of sound.

Both waves (ballistic and muzzle) are alternately registered by a microphone or a group of microphones located along the firing direction line. Based on the arrival times of the muzzle and ballistic waves at each microphone or group of microphones, the distances from the sound source to the devices registering the sound waves are determined. The velocity of the ballistic wave above a microphone or group of microphones is measured. At the same time, the initial projectile velocity is determined. At a given moment in time and at a known distance, the propagation velocity of the ballistic wave is equal to the projectile velocity. Subsequently, using the data from three points and the characteristics of the projectile motion, an approximating parabola is constructed, which determines the location of the projectile impact with the surface.

The schematic layout of the equipment is identical to that shown in **Fig. 2.6** (Section 2.6).

The detailed step-by-step procedure for determining projectile impact coordinates based on acoustic registration of ballistic and muzzle waves is provided in Section 2.6. In this chapter, the method is applied to compute trajectory approximation coefficients and mean impact coordinates under varying operational conditions.

8.4 Acoustic fields of a shot – the basis of the method for recording the projectile impact coordinate with the surface

Ensuring the required accuracy of artillery fire under modern conditions is directly associated with the capability for prompt assessment of the results of each shot. In this context, the analysis of acoustic fields generated during a shot and the subsequent projectile flight is considered one of the promising information-based approaches to solving the problem of artillery fire verification [2]. Modern means of ensuring the accuracy of artillery fire and its current assessment, as well as guidance and correction methods, can be divided into the following technological directions: (i) preliminary fire preparation [15]; (ii) verification of firing results, i.e., confirmation of projectile impact at the aiming point or assessment of deviation from the aiming point [16].

The technological direction (i) provides comprehensive accounting of possible firing errors, including systematic ones, and encompasses a set of preparatory measures, among which the following main components are distinguished: target reconnaissance and coordinate determination; topogeodetic preparation; meteorological preparation; ballistic preparation; technical preparation; and determination of firing settings. The implementation of these measures is aimed at reducing systematic firing errors prior to the execution of a shot.

Despite continuous improvement of the technological means associated with direction (i), including the use of information technologies, random disturbances may arise during firing. These disturbances are associated with factors that are difficult to assess with sufficient accuracy, such as barrel wear of the artillery system that has occurred since its last measurement; changes in barrel temperature as a result of intensive previous firing; and inaccurate information regarding the propellant charge and its storage conditions. Errors caused by random disturbances must be assessed during the verification process [17], which necessitates the use of additional information means for monitoring firing results.

Technological direction (ii) is associated with establishing informational feedback between successive shots fired from an artillery system [2] and involves estimating the coordinates of projectile bursts during firing. For fire correction, it is necessary to evaluate the coordinates of the projectile burst at the moment of firing, which significantly complicates and prolongs the verification procedure.

The most commonly used technologies within direction (ii) at present include the following:

1. Optical observation, including the use of unmanned aerial vehicles [18]. The disadvantages are demasking of the observation process and vulnerability of the observation assets.

2. Determination of the projectile impact point using artillery radar stations (Radar Stations) [19]. The disadvantage is demasking of the observation process due to radar emission.

3. Processing of acoustic signals from projectile bursts, i.e., the use of artillery sound-ranging means, specifically for "own-fire adjustment" [20]. The disadvantages include the requirement for large, spatially distributed sensor systems and a strong dependence of effectiveness on meteorological conditions.

The means of analyzing artillery acoustic fields are considered in more detail below. During an artillery shot, two types of waves are generated. A sound impulse produced by the propellant gases exiting the barrel immediately behind the projectile forms a wave known as the muzzle wave [18]. A wave with similar acoustic characteristics also arises during a projectile burst. This type of wave constitutes the object of analysis in artillery sound ranging.

Another type of wave generated during a shot is the air wave produced by the projectile moving at supersonic speed, referred to as the ballistic wave [11]. The ballistic wave remains a shock wave throughout the entire period during which the projectile continues to move at supersonic speed, while propagating together with it. The acoustic signal of the ballistic wave has an *N*-shaped impulse with a duration of 2–5 ms; its energy spectrum is broadband and lies within the frequency range from 10 Hz to 500–700 Hz. The ballistic wave can be registered only within the Mach cone formed by a projectile flying at supersonic speed. For more than 100 years in artillery sound ranging, the ballistic wave was considered an interference signal. However, in [1] it was demonstrated that the ballistic wave is a valuable source of useful information, in particular regarding the current level of barrel wear. This study examines the possibility of developing and investigating a promising method for verification of an artillery shot based on registration of the ballistic wave generated by a projectile flying along a trajectory recorded by a spatially distributed system of acoustic sensors.

8.5 Method for shot verification

General description of the shot verification method. **Fig. 8.1** presents the layout of the artillery gun and the measuring equipment employed to record the projectile flight parameters.

The origin of the coordinate system is aligned with the weapon firing position P_0, from which a projectile is fired with an initial velocity vector $\vec{v}_0$ exceeding the speed of sound. The diagram illustrates the projectile motion in the vertical plane. The aiming point P is located at the horizontal range X. It should be noted that, in this study, the lateral deviation of the projectile caused by spin drift (derivation) is not taken into account. This assumption is introduced in order to focus on the longitudinal component of the projectile motion and to simplify the analysis of the proposed verification method. The influence of derivation can be incorporated into the model using well-known correction techniques if higher accuracy in the lateral plane is required [21].

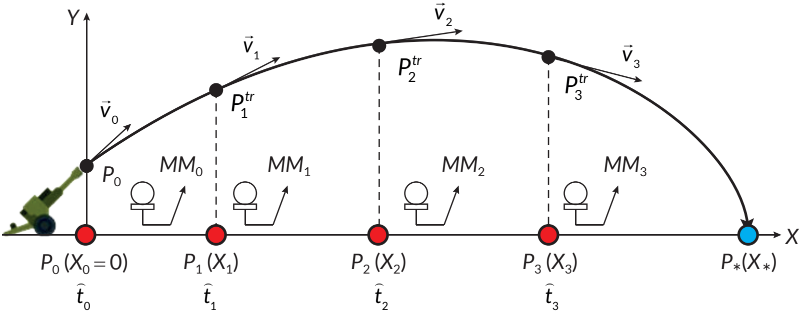


Fig. 8.1 Layout of the artillery gun and measuring equipment

During the projectile flight along a ballistic trajectory at supersonic speed, it is accompanied by a ballistic wave that is observed on the ground surface along the firing line located inside the Mach cone. On the surface, at three observation points with the corresponding coordinates $P_1(X_1)$, $P_2(X_2)$ and $P_3(X_3)$, sets of measuring equipment (Measurement Equipment Set, MES) are deployed. Each MES is intended to record the instants of occurrence of the ballistic wave at the corresponding point $\widehat{t}_i, i=1,2,3$, which correspond to the moments when the projectile passes over the observation points. Each MES includes a measuring microphone, an analog-to-digital converter, and a radio communication channel. One additional MES is located at the firing position. All four MES units are synchronized in time.

Objective of the proposed shot verification method. Taking random disturbances into account, the objective of the proposed shot verification method is to estimate the projectile impact coordinates based on the recorded moments of its passage over the observation points and to determine whether the obtained coordinates satisfy the specified accuracy requirements.

Measurement processing method:

Step 1. At the firing point P_0, the projectile velocity is determined using firing tables for the given type of projectile and propellant charge (the firing preparation is assumed to be complete, while possible random disturbances are allowed).

For each observation point P_j $(j=1,2,3)$, the following calculations are performed based on the data obtained at the preceding points P_i $(i=0,1,2)$.

Step 2. The segment of the projectile trajectory $P_i^{tr}P_j^{tr}$ is approximated by a straight-line segment $P_i^{tr}P_j^{tr}$ (**Fig. 8.2**).

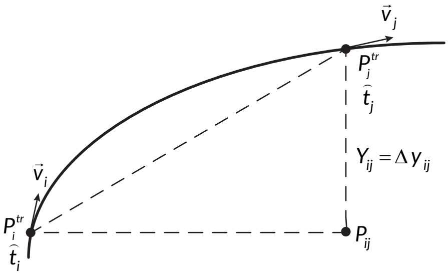


Fig. 8.2 Calculated segment of the trajectory

The air drag force acting on a projectile in flight is described by a quadratic model

$$R=C_x\rho\frac{v^2}{2}SM, \tag{8.1}$$

where C_x is the integral drag coefficient; ρ is the air density; v is the projectile velocity; S is the projectile midsection (reference) area, P_iP_j; $M=v/c$ is the Mach number; c is the speed of sound in air.

The vector $\vec{R}$ is directed opposite to the velocity vector $\vec{v}$; accordingly, the force R imparts a negative acceleration to the projectile

$$a=R/m, \tag{8.2}$$

where m is the projectile mass.

Then, the projectile velocity at point P_j^{tr} is

$$v_j=v_j-C_x\rho\frac{v^2}{2}SM/m. \tag{8.3}$$

The approximate length of the trajectory segment is

$$P_i^{tr}P_j^{tr}=v_j\Delta t_{ij}-C_x\rho\frac{v^2}{2}SM/m\Delta t_{ij}^2, \tag{8.4}$$

where $\Delta t_{ij}=t_j-t_i$.

As a result of performing the computational procedures (8.1)–(8.4), in the right triangle $P_i^{tr}P_j^{tr}P_{ij}$ two sides are known: $P_i^{tr}P_j^{tr}$ and $P_i^{tr}P_{ij} = X_j - X_i$. Thus, the elevation of the projectile flight trajectory along the *Y*-axis at point P_j^{tr} relative to point P_i^{tr} is determined

$$\Delta Y_{ij} = \sqrt{(P_i^{tr}P_j^{tr})^2 - (P_i^{tr}P_{ij})^2}\,. \tag{8.5}$$

The projectile flight height over point P_j is

$$Y_j = Y_j + \Delta Y_{ij}\,. \tag{8.6}$$

As a result of performing the described computational procedure, the points of projectile trajectory elevation over the measurement points $P_1^{tr}(X_1,Y_1)$, $P_2^{tr}(X_2,Y_2)$, $P_3^{tr}(X_3,Y_3)$ are obtained, including the firing point $P_0\ (X_0 = 0, Y_0 = 0)$.

Step 3. Approximation parabolas are constructed for each triad of points:

$$\begin{aligned}
&(P_0,P_1^{tr},P_2^{tr}) - Y_1 = A_1X^2 + B_1X,\\
&(P_0,P_2^{tr},P_3^{tr}) - Y_2 = A_2X^2 + B_2X,\\
&(P_0,P_1^{tr},P_3^{tr}) - Y_3 = A_3X^2 + B_3X.
\end{aligned} \tag{8.7}$$

One more approximating parabola can be constructed using four points

$$(P_0,P_1^{tr},P_2^{tr},P_3^{tr}) - Y_4 = A_4X^2 + B_4X\,. \tag{8.8}$$

Each of the approximating parabolas represents a model of the projectile motion that, to a certain extent, compensates for random disturbances occurring at the moment of firing. The intersection points of the approximating parabolas with the ground surface (the non-zero roots of the parabolas) $P_*^1, P_*^2, P_*^3, P_*^4$ provide approximate estimates of the projectile impact point. The arithmetic mean of the intersection points with the surface, $X_*^0 = (X_*^1 + X_*^2 + X_*^3 + X_*^4)/4$ is taken as an average estimate of the projectile landing coordinate with partially compensated random disturbances.

It should be noted that, in practice, more than four approximating parabolas are constructed; however, for the evaluation of X_*^0, exactly four are selected, in accordance with the algorithm presented below.

Algorithm for determining the signs of ΔY_{ij}:

In the calculations of relations (8.2)–(8.6), it was assumed that point *j* lies on the trajectory above point *i*. In practice, the *Y*-coordinate of point *i* may be greater than the *Y*-coordinate of point *j*. This occurs, for example, if point *j* is on the descending branch of the projectile's trajectory. Therefore, when evaluating ΔY_{ij} using expression (8.5) both positive and negative values of ΔY_{ij} should be taken into account. Accordingly, for each point $P_j, (j=1,2,3)$ two values of the projectile's flight height above point P_j must be calculated

$$Y_j^+ = Y_j + \Delta Y_{ij}; Y_j^- = Y_j - \Delta Y_{ij}. \tag{8.9}$$

Next, during the construction of approximating parabolas for each point $P_j, (j=1,2,3)$ included in the triad (8.7) or the tetrad (8.8), two parabolas are built – $Y_j^+ = A_j^+ X^2 + B_j^+ X$ and $Y_j^- = A_j^- X^2 + B_j^- X, (j=1,2,3)$. Then, for each of the two parabolas, the distance from the approximated landing point is determined as $\Delta_j^+ = \left| X_{*j}^+ - X_* \right|, (j=1,2,3)$ and $\Delta_j^- = \left| X_{*j}^- - X_* \right|, (j=1,2,3)$. The parabola with the smaller value of $\Delta_j, (j=1,2,3)$ is selected as the "correct" approximating parabola.

Accounting for the relative positions of points P_j and P_i, as ensured by the proposed algorithm, results in the formation of four "correct" approximating parabolas at Step 3 of the described method.

8.6 Simulation modeling of the shot verification method

The proposed method was validated by means of simulation modeling of firing from an FH70 howitzer using an M107 projectile of 155 mm caliber. In the simulation, it was assumed that the moments of ballistic wave registration can be determined with sufficient accuracy, while random disturbances were introduced only in the temporal parameters of projectile motion. The projectile characteristics [22] are as follows: mass 43 kg, diameter 0.15471 m, initial velocity (full charge No. 8) 684.3 m/s. Firing preparation is assumed to be complete. For the calculations, the following values were adopted: $\rho = 1.2041$ kg/m^3, $c = 341.6$ m/s, and the coordinate of the aiming point $X_* = 25000$ m. The integral drag coefficient C_x was taken from tables [22]. The *X*-coordinates of the measurement points are: $X(P_1) = 4900$ m, $X(P_2) = 10000$ m, $X(P_3) = 16000$ m. The times of projectile passage over the measurement points, taking into account random disturbances $\widehat{t}_i, i = 1,2,3$, were generated as follows: 5% random disturbances were introduced into the tabulated passage times

$$\widehat{t}_i = t_i^{FT} + \Delta t_i, \Delta t_i \in rand[0.95\Delta t_i; 1.05\Delta t_i] \, (i=1,2,3).$$

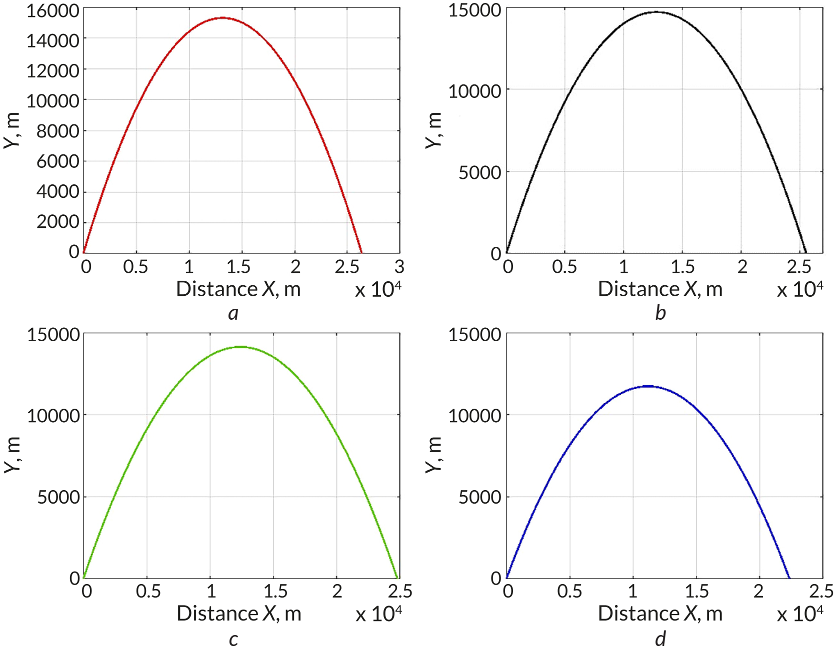
The simulated parameters of the simulation modeling are presented in **Table 8.1**.

Table 8.1 Parameters of simulation modeling of the shot verification method

Simulated parameter	Parameters corresponding to the microphone locations along the firing direction		
	No. 1	No. 2	No. 3
Target coordinate at the aiming point, m	25000	25000	25000
Distance from the gun to the measuring microphone, m	4900	10000	16000
Time of ballistic wave registration, s	10.5	22.2	39.7
Flight height over the measurement point Y_j, m	3634	6163	6686

Using the developed methodology, four approximating parabolas were constructed (**Fig. 8.3**). The results of the simulation modeling are presented in **Table 8.2**.

For clarity of the approximation process, the final segments of the approximating parabolas are shown in **Fig. 8.4**.



a *b* *c* *d*

Fig. 8.3 Approximating parabolas: *a* – through points P_0, P_1, P_2; *b* – through points P_0, P_1, P_3; *c* – through points P_0, P_2, P_3; *d* – through points P_0, P_1, P_2, P_3

Table 8.2 Results of simulation modeling of the shot verification method

Simulation results		Approximating parabolas			
		No. 1	No. 2	No. 3	No. 4
Target range X_*, m		25000	25000	25000	25000
Parabola equation coefficients $Y = AX^2 + BX$	A	–0.000092	–0.000090	–0.000090	–0.000088
	B	2.28	2.30	2.11	2.32
X_*^i, m		25070	25060	24610	22850
$\underline{X}_*$, m		24390	24390	24390	24390

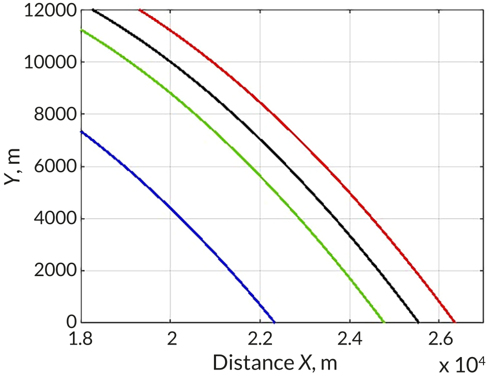


Fig. 8.4 Final sections of the approximating parabolas

Simulation modeling of the proposed shot verification method makes it possible to identify two main results:

1) the verification method, due to the use of a system of approximating parabolas, compensates for random disturbances while maintaining an error of about 0.5% of the firing range;

2) the proposed method allows verification with a single shot, providing results even before the projectile actually lands.

8.7 Comparative analysis of methods

Comparative analysis of the proposed method for registering the coordinates of an artillery projectile's impact with the surface and the method of compensating random disturbances through successive corrective shots made it possible to identify the main advantages and limitations of each approach.

In order to demonstrate the effectiveness of the proposed method, a model calculation of firing at the same range was carried out using a program for precise trajectory computation based on the NATO standard STANAG 4355 [23]. According to this standard, the projectile flight model describes the projectile as a moving material point with five degrees of freedom. At present, such a model is considered one of the most accurate for describing the trajectories of large-caliber artillery projectiles.

The complete simulation, in accordance with [23], was performed using Matlab software code presented in [24]. To ensure equivalent simulation conditions, random disturbances were taken into account by means of a pseudo-random variation of the projectile's initial velocity

$$v_0^{dist} = v_0^{FT} + \Delta v_0, \Delta v_0 \in rand[0.975 v_0^{FT}; 1.025 v_0^{FT}]. \tag{8.10}$$

After each shot, the deviation of the projectile's impact point was estimated and subsequently compensated using the artillery bracketing method with correction of the aiming angle for the subsequent shots. **Fig. 8.5** presents the calculated projectile flight trajectories for five successive shots. The generalized simulation results are given in **Table 8.3**.

For clarity, **Fig. 8.6** shows the terminal segments of the ballistic trajectories for the specified five shots.

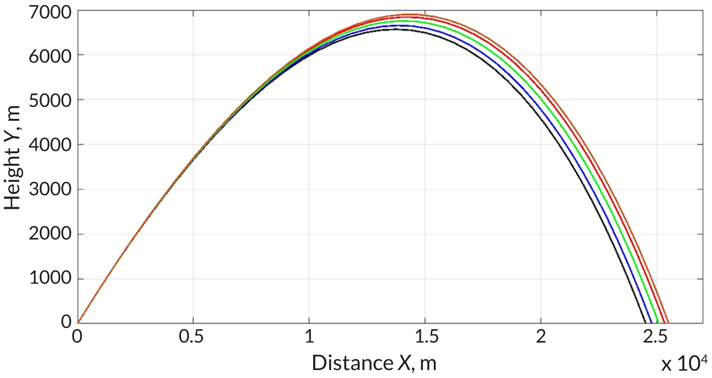


Fig. 8.5 Calculated ballistic trajectories for five shots with subsequent correction

Table 8.3 Results of modeling random disturbance compensation by successive shots using the STANAG 4355 model

Simulation results	Shot numbers				
	No. 1	No. 2	No. 3	No. 4	No. 5
Target range X_*, m	25000	25000	25000	25000	25000
X_i, m	24550	25530	24750	25300	25120
$\underline{X}_*$, m	25120	25120	25120	25120	25120

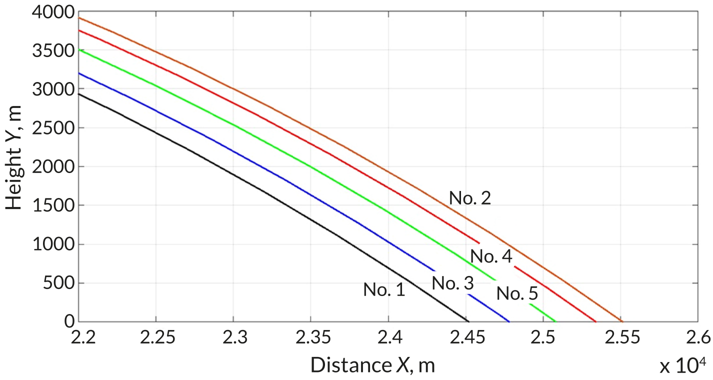


Fig. 8.6 Final segments of ballistic trajectories for five successive shots

The obtained simulation results demonstrated that, in order to compensate for random disturbances down to an error level of approximately 0.5% of the firing range, the traditional adjustment method requires at least five consecutive shots.

For practical verification of the effectiveness of the proposed shot verification method, a full-scale field experiment was conducted with registration of real ballistic wave signals during training firing of a 152 mm towed gun 2A36 "Hyacinth-B". Three sets of measurement equipment were deployed at distances of 5400 m, 7800 m, and 10 000 m from the firing position. Each set included a Rode NT-USB condenser microphone, a 16-bit TASCAM analog-to-digital converter, and radio communication channel equipment with the firing position, where an additional measurement set was installed. All measurement equipment operated in a time-synchronized mode.

Due to the high level of acoustic interference and the low signal-to-noise ratio, the ballistic wave signal was recorded in a distorted form (**Fig. 8.7**). Under such conditions, determining the moment of its onset using threshold-based methods becomes significantly more difficult. Therefore, the determination of the response time of the measuring microphones to the ballistic wave was carried out by identifying the maximum of the cross-correlation function between the signals recorded at the measurement point and at the firing position. For this purpose, the cross-correlation function of the signal $s_i(t)$ at the measurement point and the signal $s_0(t)$ at the firing position was used [25]

$$\hat{t}_i = R(s_i(t), s_0(t))\,(i = 1, 2, 3)\,. \tag{8.11}$$

A typical form of the cross-correlation function is shown in **Fig. 8.8**.

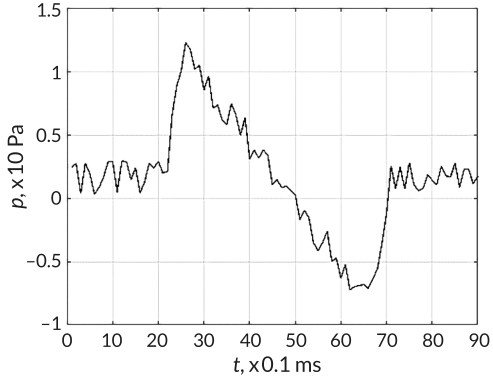


Fig. 8.7 Ballistic wave signal recorded at measurement point No. 1
Source: [26]

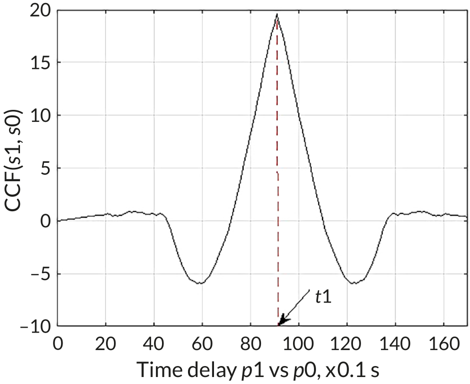


Fig. 8.8 Cross-correlation function of signals at point No. 1 and the firing position
Source: [26]

The results of the field experiment are presented in **Tables 8.4**, **8.5**. It should be noted that in this case only three approximating parabolas were constructed, each defined by three points.

During the field tests, the proposed method provided compensation of random disturbances at the level of 3.5% of the firing range. This value significantly exceeds the result obtained in the simulation experiment. However, considering that it was achieved with a single shot, this result demonstrates a quite acceptable quality of verification.

Let's turn to the identification of the advantages and disadvantages of the proposed shot verification method. The specified verification method based on parabolic approximation, as demonstrated by simulation modeling and full-scale experiments, makes it possible to compensate random disturbances to quite acceptable error levels already with a single shot [16].

Table 8.4 Parameters of the experimental verification of the shot verification method

Parameter	**Parameters corresponding to the microphone placement points along the firing line**		
	No. 1	**No. 2**	**No. 3**
Aimed target coordinate, m	15000	15000	15000
Distance from the gun to the measuring microphone, m	5400	7800	10000
Time of ballistic wave registration, s	8.5	14.8	19.7
Height of projectile over the measurement point Y_j, m	3634	6163	6686

Table 8.5 Results of the experimental verification of the shot verification method

Simulation results		**Approximating parabolas**		
		No. 1	**No. 2**	**No. 3**
Target range X_*, m		15000	15000	15000
Parabola equation coefficients $Y = AX^2 + BX$	A	–0.000096	–0.000088	–0.000080
	B	1.32	1.28	1.19
X_*^i, m		13790	14550	15100
$\underline{X}_*$, m		14480	14480	14480

The computations accompanying the construction of the approximating parabolas are fairly simple and can be performed on an artillery commander's tablet.

The recommended method reduces the time required for fire execution and decreases ammunition expenditure. In this way, it offers new opportunities for preserving weapon survivability in counter-battery operations. The main drawback of the developed verification method is the need for specialized equipment to register acoustic fields and to deploy it along the line of fire. However, information on the development of such equipment and its pilot application is currently available [27].

Based on the results presented in Chapter 4, it can be stated that, for the first time, a method for tracking an artillery shot under the influence of random disturbances has been proposed, relying on the registration of ballistic and muzzle waves generated by the shot and the projectile. This made it possible to verify the state of an artillery gun shot and to determine the point of impact of the projectile with the surface using parabolic approximation of its characteristics.

8.8 Conclusions

In this chapter of the monograph, a method for verifying an artillery shot based on the registration of acoustic fields, in particular ballistic and muzzle waves generated during the shot and the projectile's flight, has been considered and investigated. It is shown that the use of information on the time instants at which the ballistic wave passes over spatially distributed observation points makes it possible to estimate the projectile's impact coordinates even before it collides with the surface.

The proposed method is based on parabolic approximation of segments of the projectile flight trajectory and provides compensation for random disturbances associated with instability of the initial velocity, barrel condition, and other poorly controllable factors. The developed algorithm for determining the signs of trajectory elevation makes it possible to correctly account for the relative positioning of measurement points on the ascending and descending branches of the trajectory.

The results of simulation modeling confirmed the ability of the method to ensure accuracy of impact coordinate estimation at a level of about 0.5% of the firing range using a single shot. A field experiment involving the registration of real ballistic wave signals demonstrated the practical feasibility of the approach and an acceptable quality of verification under conditions of a high level of acoustic interference.

Comparative analysis with traditional adjustment methods showed that the proposed approach makes it possible to significantly reduce the number of corrective shots, decrease ammunition consumption, and shorten the time required for fire execution, which is critically important in modern counter-battery operations. The obtained results indicate the promise of further development of artillery shot verification methods based on acoustic measurements and their integration into modern artillery information and control systems.

Conflict of interest

The authors declare that they have no conflict of interest in relation to this research, whether financial, personal, authorship or otherwise, that could affect the research and its results presented in this paper.

Use of artificial intelligence statement

The authors declare that they did not use artificial intelligence tools in preparing this manuscript.

Authors' contributions

Pavlo Gultsov: Conceptualization, Methodology, Development of acoustic verification algorithms, Writing – original draft.

Viktor Boltenkov: Simulation modeling, Algorithm implementation, Analysis of projectile flight data.

Yevhenii Dobrynin: Data acquisition, Processing of acoustic signals, Validation of results.

Oleksii Maksymov: Visualization of results, Technical support of field experiments, Writing – review & editing.

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