CHAPTER 1

A strategic approach to energy-efficient methods of navigation, maneuvering and ship control

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Abstract

This article will explore the various technologies and strategies available to improve energy efficiency on ships and provide a scientific analysis of their effectiveness. The research objectives include development of a comprehensive model for assessing vessel energy efficiency in the context of navigation operations, identification and analysis of key factors affecting fuel consumption during vessel's voyage, formulation of practical recommendations for implementing energy-efficient navigation methods. An innovative approach is presented to improving vessel energy efficiency through enhanced navigation methods, addressing the growing need for fuel consumption optimization in maritime transportation. Traditional approaches to vessel energy efficiency often focus on technical solutions, while the potential for optimization through improved navigation methods remains underexplored. The study introduces a comprehensive model for energy efficiency assessment that considers multiple operational factors affecting fuel consumption during vessel transit. It is provided a systematic approach to energy efficiency optimization, supported by mathematical models and practical recommendations for implementation. The results demonstrate the potential for significant reduction in fuel consumption through improved navigation methods, contributing to both economic efficiency and environmental sustainability in maritime operations. A method for increasing the energy efficiency of a vessel by minimizing the variance of the observation error with the introduction of an orthogonal decomposition of the distribution density of errors in navigation measurements is discussed. The results obtained can significantly increase the accuracy of the vessel's location and, as a consequence, improve its energy efficiency by reducing deviations from the optimal route. The proposed method for determining the ship's coordinates using the orthogonal decomposition of the error distribution density provides higher efficiency compared to the least square method.

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Improving the energy efficiency of ships is an important step in reducing the shipping industry's impact on the environment. There are various strategies and technologies that can be employed to achieve this goal, including hull coatings, waste heat recovery, energy management systems, hybrid propulsion systems, and wind propulsion. While each strategy has its advantages and limitations, their combined use can help improve the overall energy efficiency of ships and reduce their impact on the environment.

Keywords

Energy efficiency, vessels fuel consumption, wind propulsion technologies, ballast optimization, trajectory control, navigation methods, observation, orthogonal decomposition, measurement error.

1.1 Introduction

The shipping industry is a critical component of the global economy, with around 90% of the world's trade being transported by ships. However, the industry also has a significant environmental impact, contributing to around 2.5% of global greenhouse gas emissions. As such, there is a growing need to improve the energy efficiency of ships to reduce their environmental impact and comply with increasingly stringent regulations.

Improving energy efficiency on ships can also have significant economic benefits by reducing fuel consumption and operating costs. For example, a 2018 report by the International Maritime Organization (IMO) found that implementing energy efficiency measures could result in fuel savings of up to 75%, with corresponding reductions in emissions and operating costs.

1.2 Energy efficiency on ships. Strategic approach

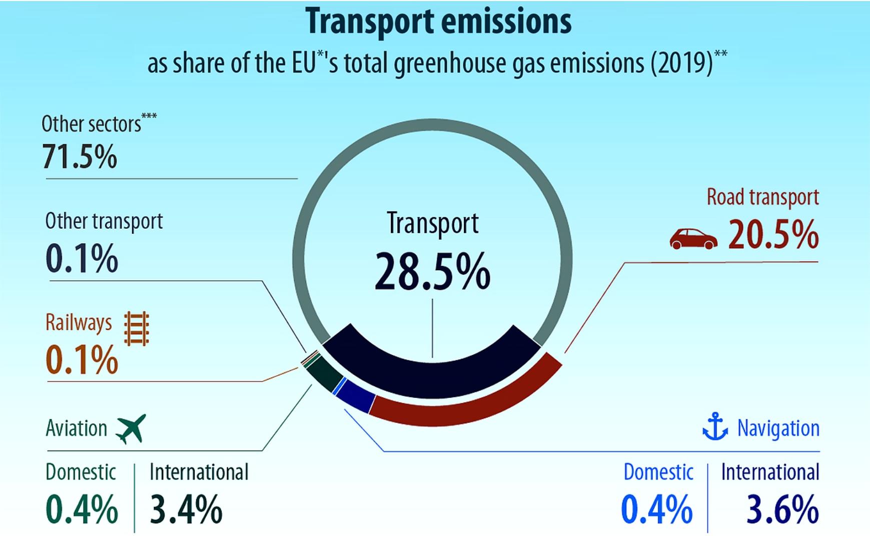
Given the growing interest in improving energy efficiency on ships, there have been significant developments in technologies and strategies to achieve this goal. These range from advanced propulsion systems to hull coatings, waste heat recovery, and energy management systems. However, there is a need for further scientific analysis to assess the effectiveness of these technologies and strategies in different ship types and operating conditions, as well as to identify new solutions for improving energy efficiency in the shipping industry.

The shipping industry is a significant contributor to global greenhouse gas emissions, accounting for around 2% of global emissions (around 4% in EU) in 2019.

The primary source of emissions from ships is the burning of fossil fuels, which power the large diesel engines that propel the vessel through water (the shipping emission output varies on ship's types and their routes) (**Fig. 1.1, 1.2**).

The need to improve energy efficiency on ships is becoming increasingly urgent due to global efforts to address climate change. The IMO, a United Nations agency responsible for regulating shipping, has set targets to reduce greenhouse gas emissions from the shipping industry. These include a target to reduce the carbon intensity of international shipping by at least 40% by 2030, compared to 2008 levels, and to reduce total greenhouse gas emissions from international shipping by at least 50% by 2050, compared to 2008 levels.

Improving energy efficiency on ships can also have significant economic benefits. Fuel costs can account for up to 60% of a ship's operating costs, and reducing fuel consumption can result in substantial savings, plus as per post-pandemic and war/post-war circumstances fossil fuel price will continue to grow (despite fuel spread and availability) (**Fig. 1.3**).



**Excluding the United Kingdom*

***Excluding land use, land-use change and forestry*

****Energy, industry, residential, commercial, institutional, agriculture, forestry, fisheries and other*

Fig. 1.1 Greenhouse gas emissions in EU, 2019
Source: [1]

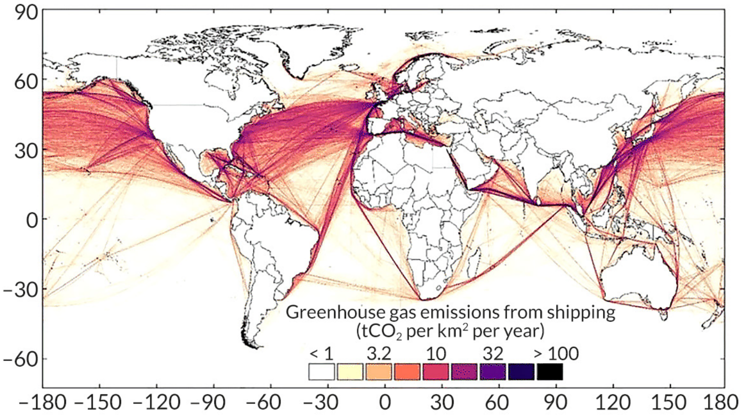


Fig. 1.2 Global distribution of gas emissions from shipping
(as per routes/traffic density, AIS)
Source: [2]

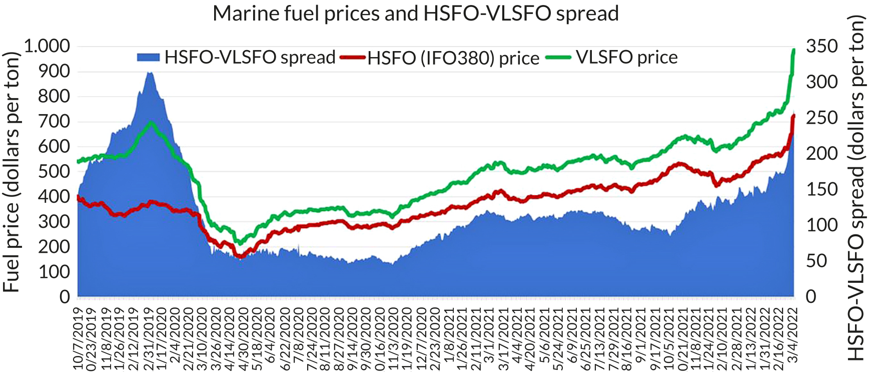


Fig. 1.3 HSFO/VLSFO prices worldwide, 2019–2022
(chart by "American Shipper" based on data from Ship & Bunker)
Source: [3]

Therefore, there is a growing need to identify and implement technologies and strategies to improve energy efficiency on ships. These can range from simple measures such as optimizing vessel speed and route planning to more advanced solutions such as hybrid or electric propulsion, hull coatings, waste heat recovery, and energy management systems.

There are several technologies and strategies that can be used to improve energy efficiency on ships, including but not limited to:

1. Hull coatings can be used to reduce drag and improve the flow of water around the ship's hull, thereby reducing fuel consumption. For example, a silicone-based coating can reduce frictional resistance by up to 20%, while a super-hydrophobic coating can reduce drag by up to 30% [4]. A study conducted by the IMO estimated that a silicone-based hull coating could result in fuel savings of up to 5%, depending on the ship's size and operating conditions. However, the effectiveness of hull coatings depends on the specific design and operating conditions of the ship, and some hull coatings may not be effective in all situations.

2. Waste heat recovery systems can capture and reuse heat generated by the ship's engine, reducing the amount of fuel needed to generate the same amount of power [5]. For example, a waste heat recovery system can recover up to 40% of the energy from the engine's exhaust gas. A study conducted by the European Commission estimated that a waste heat recovery system could result in fuel savings of up to 10%, depending on the ship's size and operating conditions. There are various waste heat recovery technologies available, including steam turbines, organic Rankine cycles, and thermoelectric generators.

3. Energy management systems can be used to optimize the operation of the ship's engines and equipment, reducing energy waste and improving efficiency [6]. These systems can monitor and control fuel consumption, adjust engine speed and power, and optimize equipment performance. A study conducted by the IMO estimated that an energy management system could result in fuel savings of up to 15%, depending on the ship's size and operating conditions. Energy management systems can also help reduce maintenance costs and extend the lifespan of the ship's equipment.

ISO 50001 is a voluntary international standard designed to be compatible and harmonized with other system standards, such as ISO 14001 for environmental management systems and ISO 9001 for quality management systems. It applies to organizations of any size, and provides requirements for establishing, managing and improving their energy consumption and efficiency. Certification to the standard can contribute to:

– improvement of energy performance, including energy efficiency, use and energy consumption;

– reducing environmental impact, including greenhouse gas emissions without affecting operations and simultaneously increasing profitability;

– continual improvement of the energy management systems;

– ensuring measurement, documentation, reporting and benchmarking of energy consumption;

– credible market communication about energy performance efforts.

Therefore, the abovementioned principles can be applied in developing various strategies for energy efficiency on ships directly, as these are something that navigators can control and maintain on the operational level.

4. Hybrid propulsion systems use a combination of conventional and electric power to reduce fuel consumption and emissions. For example, a ship equipped with a hybrid propulsion system can switch between diesel and electric power depending on the operating conditions, such as when entering and leaving ports. A study conducted by the European Commission estimated that a hybrid propulsion system could result in fuel savings of up to 30%, depending on the ship's size and operating conditions [6]. Hybrid propulsion systems can also help improve the overall performance and reliability of the ship's power system.

5. Wind propulsion systems can be used to harness the power of wind to reduce fuel consumption. These systems can range from simple sails to more advanced solutions such as rotor sails or kites [7]. A study conducted by the University of Delaware estimated that a rotor sail system could result in fuel savings of up to 10%, depending on the ship's size and operating conditions. While wind propulsion technologies are still relatively new, they have shown promising results in terms of reducing fuel consumption and emissions.

1.3 Increasing energy efficiency by improving navigation methods

The optimization of vessel energy efficiency represents a critical challenge in modern maritime transportation, driven by both economic pressures and environmental concerns. As global shipping continues to grow, the need for effective methods to reduce fuel consumption while maintaining operational safety and efficiency becomes increasingly important. Traditional approaches to energy efficiency improvement have primarily focused on technical solutions and vessel design modifications [8], while the potential for optimization through enhanced navigation methods remains insufficiently explored.

Recent studies have highlighted the significant impact of navigation practices on vessel fuel consumption. The relationship between vessel routing, weather conditions, and energy efficiency has been extensively documented, yet comprehensive approaches to optimizing these factors through navigation methods are limited. Current research indicates that proper navigation strategy can significantly affect fuel consumption, particularly in challenging weather conditions and confined waters [9].

The International Maritime Organization's regulations on vessel energy efficiency have created additional pressure for developing more sophisticated approaches to fuel consumption optimization. While existing studies have addressed various aspects of this challenge, a comprehensive methodology integrating multiple operational factors remains lacking. This gap is particularly evident in the context of real-time navigation decision-making and its impact on energy efficiency.

The core methodology centers on the energy efficiency criterion Q, which represents the total fuel consumption required for vessel transit. The general form of this criterion is expressed as

$$Q = q \times D \times S,$$

where q represents specific fuel consumption per ton-mile at average specified speed; D denotes vessel displacement; S indicates route length. The following requirements must be met

$$P < \left[P\right] \text{ and } S' > S,$$

where P – the accident probability; S' – the actual route length. In steady-state motion, the following expression is valid

$$V = \text{const or } Te = kV^2, \text{ i.e., } V = V\left(Te/k\right),$$

where V – the vessel's steady-state speed; Te – the propeller thrust force; k – the movement resistance coefficient.

The propeller thrust force Te, under otherwise equal conditions, depends on the current fuel consumption Gr, average draft T, and vessel trim ψ, i.e., $Te = f\left(Gr, T, \psi\right)$. In turn, coefficient k can be represented by two components: k_1 – hull resistance and k_2 – rudder blade resistance when deflected from the centerline. Component k_1 depends on vessel trim ψ, and k_2 depends on rudder blade steering frequency n, therefore $k = f(y, n)$.

Consequently

$$V = f\left(Gr, T, \psi, n\right).$$

The vessel's displacement D is by definition the sum of the light ship weight Po, ship stores weight Pz, cargo weight Pg, and ballast weight Pb, where

$$D = Po + Pz + Pg + Pb.$$

The actual route length S' is the sum of the planned route length S and additional distance losses $\Delta\delta$ caused by vessel drift relative to the programmed movement trajectory under external disturbances, i.e., $S'=\delta+\Delta\delta$.

Considering the above, it is possible to write

$$Q=\left(P_o+Pz+Pg+P'b\right)\left(S+\Delta S\right)q',$$

where q' – the weighted average fuel consumption that can take values $G'r$ different from Gr depending on the ratio of speeds V and Vz.

The values Po, Pz, Pg, and δ for a specific transit are constant parameters, while $P'b$, $\Delta\delta$, and q' are controlled variables.

Therefore, the general expression for the energy efficiency criterion Q can be written as

$$Q=f\left(P'b,\Delta\delta,q'\right).$$

Thus, based on the above, the task of improving energy efficiency can be formalized as an optimization problem, with Q chosen as the optimality criterion. The optimization problem itself takes the following form

$$Q\rightarrow\min;\ P<\left[P\right],$$

which takes into account the navigation safety constraint.

Analyzing the last expression, it is possible to conclude that minimization of the energy efficiency criterion Q is possible by solving three main tasks:

1. Minimization of required ballast $P'b$ for the transit.
2. Minimization of additional distance losses $\Delta\delta$.
3. Minimization of specific fuel consumption Gr through the relationship between propeller thrust force and vessel movement resistance.

These three directions define the conceptual basis for optimizing energy efficiency through improved navigation methods.

Development of the first approach may be relevant for container ships, where proper cargo placement during loading can minimize ballast intake to ensure vessel trim and stability.

The second approach involves minimizing trajectory error, which characterizes the difference between programmed and actual vessel movement trajectories, depending on the strategy for keeping the vessel on the programmed trajectory and the accuracy of position control.

The third approach allows changing the vessel's movement resistance, particularly through trim selection and rudder steering frequency, thereby achieving reduced specific fuel consumption.

The initial optimization problem can be represented as three independent component optimization problems for minimizing ballast quantity, distance losses, and specific fuel consumption.

The relationship between the average specified transit speed Vz and the vessel's steady-state speed V characterizes the current fuel consumption value. If $V > Vz$, then fuel consumption Gr can be reduced to value $G'r$, at which equality $V = Vz$ is achieved.

Thus, the research methodology incorporates three primary components:

1. Ballast optimization analysis. The study examines the relationship between ballast requirements and vessel efficiency, considering:

– static moments along longitudinal and vertical axes;
– cargo placement optimization;
– minimum ballast requirements for safe operation.

2. Trajectory control assessment. Analysis of factors affecting vessel trajectory, including:

– environmental impact on vessel movement;
– deviation from programmed route;
– optimal course correction strategies.

3. Fuel consumption optimization. Evaluation of specific fuel consumption based on:

– propeller thrust force relationships;
– movement resistance coefficients;
– impact of trim and rudder frequency.

Data collection involved analysis of operational records and theoretical modeling of vessel behavior under various conditions. The methodology employed both deterministic and probabilistic approaches to account for the dynamic nature of maritime operations [9].

It is possible to examine in more detail the possibilities for solving the three listed problems. The first component optimization problem of minimizing the required ballast for transit requires determining the necessary static moments Mx and My along the vessel's longitudinal and vertical axes, respectively, which ensure the required trim, longitudinal strength, and stability of the vessel. Cargo placement is performed in such a way that the resulting static moments are as close as possible to the required Mx and My. The static moments are brought to the necessary values by taking in minimal ballast.

As a first approximation, this problem can be formalized as follows. As previously indicated, let's choose the amount of ballast taken during container loading as the optimality criterion. Therefore, the optimality criterion is $I = Pb$, and the optimization problem itself is formalized as follows

$$Pb \rightarrow \min;\ Mx_1 < Mx < Mx_2;\ My_1 < My < My_2;\ Fi < [Fi],$$

where the second, third, and fourth lines are constraints on static moments according to seaworthiness requirements, and the last inequality limits the magnitude of the inertial force occurring during vessel rolling.

The following notations are used in the last expression:

– m and n – the number of ballast tanks and loaded holds respectively;

– Mx_1, Mx_2, My_1, My_2 – limiting boundary values of static moments satisfying seaworthiness requirements;

– Pg_i, xg_i, yg_i, zg_i – amount of cargo in the i-th cargo space and coordinates of its center of gravity;

– Pb_i, xb_i, $yb_i i$, zb_i – amount of ballast in the i-th ballast tank and coordinates of its center of gravity;

– Fi and $[Fi]$ – respectively, the largest inertial force acting on each loading container and the permissible value of inertial force.

To solve this optimization problem, it is first necessary to form a set Ω_1 of permissible loads satisfying seaworthiness constraints, and then from the obtained set, identify a subset Ω_2 satisfying the last constraint on arising inertial forces, where $\Omega_2 \subset \Omega_1$. Finally, from subset Ω_2, the optimal load is selected where the optimality criterion $I = Pb$ takes the minimum value.

Upon more detailed analysis of the optimization problem, it appears that there may be a subset of loads where the optimization problem constraints are achieved by container placement without taking ballast. Consequently, the adopted optimality criterion loses meaning, and as an optimality criterion, one can propose the maximum inertial force Fi acting on containers during vessel rolling, which should be minimized.

When determining inertial forces, one should consider the relationship between vessel loading and its rolling parameters, which determine the characteristics of arising angular accelerations and affect the magnitude of inertial forces.

The next task is minimizing distance losses, considering that these losses occur for two main reasons. First, the length of the actual trajectory is greater than the programmed one due to external disturbances, and second, during the voyage, dangerous approach situations arise and the vessel performs an avoidance maneuver, replacing a section of the programmed trajectory with an avoidance trajectory, leading to distance loss.

Consequently, the distance losses ΔS have two components ΔS_1 and ΔS_2. Component ΔS_1 determines distance losses due to the difference between programmed and actual trajectory lengths and is characterized by trajectory error, which is stochastic. Therefore, it is necessary to find the dependence of trajectory error on the parameters of the strategy for keeping the vessel on the programmed movement trajectory. The optimization problem involves selecting control strategy parameters that minimize the variance of trajectory error. If to denote D as the variance of trajectory error and u as the strategy for keeping the vessel on the programmed movement trajectory, then the task of minimizing component ΔS_1 takes the following form

$$D \to \min; u = u^* \in U,$$

where u^* and U – the optimal strategy and set of permissible strategies, respectively.

Let's consider component ΔS_2, where the vessel deviates from the programmed movement trajectory to perform an avoidance maneuver and then returns to it after completing the maneuver. Minimization of this component is achieved if the difference ΔS_i between the lengths of the programmed section S_i and the corresponding avoidance trajectory S_i' is minimal when performing each maneuver, i.e., $\Delta S_i = S_i' - S_i$. The avoidance trajectory has deviation and return sections to the programmed trajectory, and its length is always greater than the programmed trajectory section connecting the ends of the avoidance trajectory. The unchanging requirement for the avoidance trajectory is that the distance of closest approach between the vessel and the target must not exceed the maximum permissible value. The value ΔS_i depends on avoidance maneuver parameters, i.e., time and course of deviation, as well as time and course of turn to return to the programmed movement trajectory. If to denote K as the avoidance maneuver parameters, then obviously $\Delta S_i = f(K)$, and the task of minimizing component ΔS_2 of distance loss can be formalized as follows

$$\Delta S \to \min; D_{\min} > [D]; K \in \Omega,$$

where Ω – the set of permissible maneuver parameters; $D_{\min}$ and $[D]$ – the distance of closest approach and its maximum permissible value, respectively.

The third task represents minimization of specific fuel consumption Gr through the relationship between propeller thrust force and vessel movement resistance, which depends on vessel trim and rudder blade steering frequency for keeping the vessel on the programmed trajectory. The vessel speed must not be less than the average speed Vz planned for the transit. In general, it is necessary to experimentally determine the dependence of specific fuel consumption $G'r$ on vessel trim, rudder blade steering frequency, and vessel speed, i.e., $G'r = f(y, n, V)$. Then, optimal values of trim ψ^* and n^* are selected, minimizing $G'r$ under the condition that the actual vessel speed V' is not less than Vz. This task can be presented as follows

$$G'r \to \min;\ V' > Vz;\ \psi^* \in [\psi];\ n^* \in [n].$$

Thus, improving energy efficiency through the development of more advanced navigation methods can be accomplished through four considered directions by solving the corresponding optimization problems presented in general form.

It should be noted that rudder blade steering frequency simultaneously affects both the accuracy of programmed trajectory implementation and movement resistance, which must be taken into account when solving optimization problems.

1.4 Improving energy efficiency by minimizing observation error variance

In modern maritime transport operations, the issue of improving the energy efficiency of vessels has become particularly relevant [10, 11]. One of the key factors influencing energy efficiency is the accuracy of the vessel's position determination [12]. Inaccuracies in observation lead to deviations from the optimal route and, consequently, to increased fuel consumption [13]. Therefore, the development of methods to enhance observation accuracy is an important scientific and practical challenge.

A significant number of scientific studies have been devoted to improving the accuracy of vessel positioning. In [14], methods for assessing the reliability of navigation and the application of orthogonal decomposition of the error distribution density in navigation measurements are considered. Study [15] focuses on measurement processing methods and the statistical interpretation of their effectiveness. Paper [16] examines the identification of navigation error distribution laws using mixed distributions of two types. However, the issue of improving overall energy efficiency by minimizing the dispersion of observation errors still requires further research [17–19].

As shown in [14], the system of likelihood equations takes the form

$$\begin{cases} \sum_{i=1}^{n} \sin\alpha_i \dfrac{\partial}{\partial \xi_i} \ln\left[f_i(\xi_i)\right] = 0, \\ \sum_{i=1}^{n} \cos\alpha_i \dfrac{\partial}{\partial \xi_i} \ln\left[f_i(\xi_i)\right] = 0, \end{cases} \qquad (1.1)$$

$$\xi_i = X\sin\alpha_i + Y\cos\alpha_i - r_i,$$

where n – number of position lines; α_i and r_i – the direction of the gradient and the transfer of the i-th position line; f_i – density of error distribution of navigation measurements.

Considering that in the general case the correct ratio is

$$\frac{\partial}{\partial \xi_i}\ln\left[f_i(\xi_i)\right]=\frac{\dfrac{\partial}{\partial \xi_i}f_i(\xi_i)}{f_i(\xi_i)},$$

therefore, taking into account the above equality, the system of likelihood equations (1.1) takes the following form

$$\begin{cases}\sum\limits_{i=1}^{n}\sin\alpha_i\dfrac{\dfrac{\partial}{\partial \xi_i}f_i(\xi_i)}{f_i(\xi_i)}=0,\\ \sum\limits_{i=1}^{n}\cos\alpha_i\dfrac{\dfrac{\partial}{\partial \xi_i}f_i(\xi_i)}{f_i(\xi_i)}=0,\end{cases}\qquad(1.2)$$
$$\xi_i=X\sin\alpha_i+Y\cos\alpha_i-r_i.$$

The study [10] proposes using an orthogonal decomposition as the probability density function of navigation measurement errors. It is shown that the best agreement with the histograms of measurement errors is achieved by using only the first term of the orthogonal decomposition, which has the following analytical expression

$$f_i(\xi_i)=(2\pi)^{-1/2}\sigma_i^{-1}\exp\left(-\xi_i^2/2\sigma_i^{-1}\right)\left\{1+\frac{\left(\mu_{4i}/\sigma_i^4-3\right)}{24!}\left[\left(\xi_i/\sigma_i^2\right)^4-6\left(\xi_i/\sigma_i^2\right)^2+3\right]\right\},$$

where μ_{4i}/σ_i^4-3 (kurtosis of the distribution); μ_{4i} – the fourth central point of error; σ_i^2 – error variance.

Let's find the expression of the first derivative $\partial/\partial\xi_i \ln f_i(\xi_i)$ for the given orthogonal decomposition:

$$\frac{\partial}{\partial \xi_i}\ln f_i(\xi_i)=\frac{\partial}{\partial \xi_i}\ln\left\{\begin{array}{l}(2\pi)^{-1/2}\sigma_i^{-1}\exp\left(-\xi_i^2/2\sigma_i^2\right)\times\\ \times\left[1+\dfrac{\left(\mu_{4i}/\sigma_i^4-3\right)}{24!}\left[\left(\xi_i/\sigma_i^2\right)^4-6\left(\xi_i/\sigma_i^2\right)^2+3\right]\right]\end{array}\right\},$$

or

$$\frac{\partial}{\partial \xi_i}\ln f_i(\xi_i)=-\xi_i/\sigma_i^2+\left\{\frac{\left(\mu_4/\sigma^4-3\right)}{4!}\left[4\xi_i^3/\sigma_i^8-12\xi_i/\sigma_i^4\right]\right\}\Big/Q,$$

where

$$Q=\left[1+\frac{\left(\mu_4/\sigma^4-3\right)}{4!}\left[\left(\xi_i/\sigma_i^2\right)^4-6\left(\xi_i/\sigma_i^2\right)^2+3\right]\right].$$

Substituting this expression into the original system of equations, it is possible to obtain

$$\begin{cases}
\sum\limits_{i=1}\sin\alpha_i\left\{-\dfrac{\xi_i}{\sigma_i^2}+\dfrac{\dfrac{\left(\mu_{4i}/\sigma_i^4-3\right)}{(4)!}\left[4\xi_i^3/\sigma_i^8-12\xi_i/\sigma_i^4\right]}{\left[1+\dfrac{\left(\mu_{4i}/\sigma_i^4-3\right)}{(4)!}\left[\left(\xi_i/\sigma_i^2\right)^4-6\left(\xi_i/\sigma_i^2\right)^2+3\right]\right]}\right\}=0;\\
\sum\limits_{i=1}\cos\alpha_i\left\{-\dfrac{\xi_i}{\sigma_i^2}+\dfrac{\dfrac{\left(\mu_{4i}/\sigma_i^4-3\right)}{(4)!}\left[4\xi_i^3/\sigma_i^8-12\xi_i/\sigma_i^4\right]}{\left[1+\dfrac{\left(\mu_{4i}/\sigma_i^4-3\right)}{(4)!}\left[\left(\xi_i/\sigma_i^2\right)^4-6\left(\xi_i/\sigma_i^2\right)^2+3\right]\right]}\right\}=0;\\
\xi_i=x\sin\alpha_i+y\cos\alpha_i-r_i.
\end{cases}$$

In the case of equidistant position lines $\sigma_i^2=\sigma^2$ and $\mu_{4i}=\mu_4$ the previous system of equations takes the following form

$$\begin{cases}
\sum\limits_{i=1}\sin\alpha_i\left\{-\dfrac{\xi_i}{\sigma^2}+\dfrac{\dfrac{\left(\mu_4/\sigma^4-3\right)}{(4)!}\left[4\xi_i^3/\sigma^8-12\xi_i/\sigma^4\right]}{\left[1+\dfrac{\left(\mu_4/\sigma^4-3\right)}{(4)!}\left[\left(\xi_i/\sigma^2\right)^4-6\left(\xi_i/\sigma^2\right)^2+3\right]\right]}\right\}=0;\\
\sum\limits_{i=1}\cos\alpha_i\left\{-\dfrac{\xi_i}{\sigma^2}+\dfrac{\dfrac{\left(\mu_4/\sigma^4-3\right)}{(4)!}\left[4\xi_i^3/\sigma^8-12\xi_i/\sigma^4\right]}{\left[1+\dfrac{\left(\mu_4/\sigma^4-3\right)}{(4)!}\left[\left(\xi_i/\sigma^2\right)^4-6\left(\xi_i/\sigma^2\right)^2+3\right]\right]}\right\}=0;\\
\xi_i=x\sin\alpha_i+y\cos\alpha_i-r_i.
\end{cases}\qquad(1.3)$$

Solving this system of equations and having the value of the variance σ^2 and the fourth central moment μ_4 of the output error distribution, the observed coordinates of the vessel are determined without using the expression for the probability density function of the position line errors.

To estimate the efficiency, it is necessary to compute the improper integrals *q*, *p*, and *s* using the expressions provided in [15]

$$q=\int_{R1} f(x)\left\{\frac{\left[\frac{\partial^2}{\partial x^2}\varphi(x)\right]\varphi(x)-\left[\frac{\partial^2}{\partial x^2}\varphi(x)\right]^2}{\varphi^2(x)}\right\}dx;\ p=\int_{R1} f(x)\left\{\left[\frac{\frac{\partial}{\partial x}\varphi(x)}{\varphi(x)}\right]^2\right\}dx;$$

$$s=\int_{R1}\frac{\left[\frac{\partial}{\partial x}f(x)\right]^2}{f(x)}dx.$$

Let's assume there are *n* position lines which errors follow a distribution that differs from the Gaussian law – for example, a mixed distribution of the first type [16] with density function $f_1(x)$. Let's estimate the accuracy of the vessel's coordinates when they are computed using the method that applies the orthogonal expansion of the density function.

To eliminate scale parameters in the density function $f_1(x)$ and its expansion $\varphi(x)$ and ensure their compatibility, let's consider their corresponding normalized density functions $g_1(x)$ and $\psi(x)$, where *x* – a standardized and centered random measurement error. In this case

$$g_1(x)=\frac{B_1}{\left(x^2/(2n-1)+1\right)^{n+1}}. \tag{1.4}$$

Here $B_1=\dfrac{2^{2n}\left[(n)!\right]^2}{(2n-1)^{1/2}\pi(2n)!}$ – normalizing factor, *n* – significant integer parameter. The variance of *x* is 1, and the fourth central moment is

$$\mu_4=\frac{(2n-1)^2 n!\left[2(n-2)\right]!24}{2(2n)!(n-2)!}.$$

Orthogonal density decomposition $\psi(x)$ of a normalized random variable with unit variance has the following form

$$\psi(x)=\frac{1}{\sqrt{2\pi}}\exp\left(-x^2/2\right)\left[1+\frac{(\mu_4-3)}{24}\left(x^4-6x^2+3\right)\right].$$

For convenience, let's denote $f_N(x)=\frac{1}{\sqrt{2\pi}}\exp\left(-x^2/2\right)$, therefore

$$\psi(x)=f_N(x)\left[1+\frac{(\mu_4-3)}{24}\left(x^4-6x^2+3\right)\right]. \tag{1.5}$$

Efficiency e_R of the ship coordinates in this case is determined by the expression

$$e_R=\frac{q^2}{ps}, \tag{1.6}$$

where p, q and s – improper integrals that depend on densities $g_1(x)$ and $\psi(x)$.

Let's write the expressions of the nonproprietary integrals p, q and s depending on the densities $g_1(x)$ and $\psi(x)$

$$q=\int\limits_{R1} g_1(x)\left\{\frac{\left[\frac{\partial^2}{\partial x^2}\psi(x)\right]}{\psi(x)}\right\}dx-p;\ p=\int\limits_{R1} g_1(x)\left\{\left[\frac{\frac{\partial}{\partial x}\psi(x)}{\psi(x)}\right]^2\right\}dx;\ s=\int\limits_{R1}\frac{\left[\frac{\partial}{\partial x}g_1(x)\right]^2}{g_1(x)}dx.$$

Calculating the value of the integral p requires an expression for the derivative $\frac{\partial}{\partial x}\psi(x)$

$$\frac{\partial}{\partial x}\psi(x)=f_N(x)\left\{-x+\frac{(\mu_4-3)}{24}\left[-x^5-10x^2-15x\right]\right\}.$$

Therefore, the improper integral p is calculated using the expression

$$p=\int\limits_{R1}\frac{B_1}{\left(x^2/(2n-1)+1\right)^{n+1}}\left\{\frac{-x+\frac{(\mu_4-3)}{24}\left[-x^5-10x^2-15x\right]}{1+\frac{(\mu_4-3)}{24}\left(x^4-6x^2+3\right)}\right\}^2 dx. \tag{1.7}$$

To calculate the value of the integral q, an expression $\frac{\partial^2}{\partial x^2}\psi(x)$ for the second derivative should be found

$$\frac{\partial^2}{\partial x^2}\psi(x)=f_N(x)\left[Q(x)+\frac{(\mu_4-3)}{6}\left(-2x^4+9x^2-3\right)\right],$$

where

$$Q(x)=\left(x^2-1\right)\left[1+\frac{\left(\mu_4-3\right)}{24}\left(x^4-6x^2+3\right)\right].$$

Therefore

$$q=\int_{R1}\frac{B_1}{\left(x^2/(2n-1)+1\right)^{n+1}}\left\{\frac{Q(x)+\frac{\left(\mu_4-3\right)}{6}\left(-2x^4+9x^2-3\right)}{1+\frac{\left(\mu_4+3\right)}{24}\left(x^4-6x^2+3\right)}\right\}dx-p. \tag{1.8}$$

Let's find an expression for the nonproprietary integral *s*, for which it is possible to use the expression of the standard density of the unnormalized error, which has the form

$$f_1(\xi)=\frac{A_m}{\left(\xi^2/2+\lambda\right)^{m+1}},$$

where

$$A_m=\frac{2^{2m}(m!)^2}{\sqrt{2\pi}(2m)!}\lambda^{m+1/2}. \tag{1.9}$$

In [14] it is shown that for a given density the improper integral s takes the following form

$$s=\frac{(n+1)(2n+1)}{(2n-1)(n+2)}. \tag{1.10}$$

An evaluation of the effectiveness e_R was carried out for density $g_1(x)$ with values of the essential parameter equal to 2, 3, 4, 5, 6, 8, 10. **Table 1.1** shows the values of the normalizing factor B_1 and the fourth central moment μ_4 for the specified values of the parameter *n*. The calculation of the improper integrals *p* and *q* was carried out according to expressions (1.7) and (1.8) by Simpson's method with integration limits from –6 to 6, which includes all normalized and centered random variables. The integral *s* was calculated according to formula (1.10). Evaluation of efficiency e_R was carried out using expression (1.6) and its value is given in the second line of **Table 1.1**.

The paper [14] presents the results of calculating the efficiency e_G of the vessel coordinates in the case of the distribution of position line errors according to the mixed law of the first type with the density $g_1(x)$, and the calculation of coordinates was performed by the least squares method. Efficiency value e_G are also given in **Table 1.2**.

Table 1.1 Value of the normalizing factor B_1 and the moment μ_4

n	2	3	4	5	6	8	10
B_1	0.4903	0.4558	0.4402	0.4314	0.4257	0.4187	0.4147
μ_4	9	5	4.2	3.857	3.667	3.462	3.353

Table 1.2 Efficiency e_G and e_R distribution density $g_1(x)$

n	3	4	5	6	8	10
e_G	0.893	0.934	0.955	0.968	0.980	0.987
e_R	0.994	1	1	1	1	1

Analysis of **Table 1.2** shows high efficiency e_R of the ship's coordinates obtained by the proposed method of applying orthogonal decomposition, which exceeds the efficiency e_G coordinates calculated by the method of least squares.

Let's consider the situation when position line errors are distributed according to a mixed law of the second type [16] with density $f_2(\xi)$. Let's find an expression for the purpose of evaluating the efficiency of the ship's coordinates based on its calculation using equations (1.3) with the use of orthogonal density decomposition.

As in the previous case, let's use normalized densities $g_2(x)$ and $\psi(x)$, where x – normalized and centered random measurement error. Moreover

$$g_2(x)=\frac{B_2}{\left(x^2/2n+1\right)^{n+3/2}}.$$

In this expression $B_2=\dfrac{(2n+1)!}{(2n)^{1/2}2^{2n+1}(n!)^2}$ – normalizing factor, and the central fourth moment $\mu_4=\dfrac{n^2 24(n-2)!}{8n!}$.

Expressions of the improper integrals p, q and s depending on the densities $g_2(x)$ and $\psi(x)$ have the form

$$q=\int_{R1} g_2(x)\left\{\frac{\left[\frac{\partial^2}{\partial x^2}\psi(x)\right]}{\psi(x)}\right\}dx-p;\ p=\int_{R1} g_2(x)\left\{\left[\frac{\frac{\partial}{\partial x}\psi(x)}{\psi(x)}\right]^2\right\}dx;\ s=\int_{R1}\frac{\left[\frac{\partial}{\partial x}g_2(x)\right]^2}{g_2(x)}dx.$$

Taking into account the previously obtained expressions for the derivatives $\frac{\partial}{\partial x}\psi(x)$ and $\frac{\partial^2}{\partial x^2}\psi(x)$ improper integrals *p* and *q* are calculated using expressions:

$$p=\int\limits_{R1}\frac{B_2}{x^2/(2n+1)^{n+3/2}}\left\{\frac{-x+\frac{(\mu_4-3)}{24}\left[-x^5-10x^2-15x\right]}{1+\frac{(\mu_4-3)}{24}\left(x^4-6x^2+3\right)}\right\}^2 dx, \tag{1.11}$$

$$q=\int\limits_{R1}\frac{B_2}{x^2/(2n+1)^{n+3/2}}\left\{\frac{Q(x)+\frac{(\mu_4-3)}{6}\left(-2x^4+9x^2+3\right)}{1+\frac{(\mu_4-3)}{24}\left(x^4-6x^2+3\right)}\right\}^2 dx-p. \tag{1.12}$$

Similarly to the previous case, in [14], a solution of the improper integral *s* was obtained, which has the following form

$$s=\frac{(2n+3)(n+1)}{2n(n+5)}. \tag{1.13}$$

For density $g_2(x)$ efficiency e_R was calculated, and the value of the significant parameter is 2, 4, 6, 8, 10. **Table 1.3** shows the value of the normalizing factor B_2 and the fourth central moment μ_4 for the listed values of the parameter *n*.

Table 1.3 Value of the normalizing factor B_2 and the moment μ_4

n	2	4	6	8	10
B_2	0.4688	0.4350	0.4233	0.4173	0.4137
μ_4	6	4	3.6	3.43	3.33

According to expressions (1.11) and (1.12), the improper integrals p and q were calculated by Simpson's method within the limits of integration from –6 to 6. The integral s was calculated by formula (1.13). Using expression (1.6), the efficiency estimate was calculated, and its obtained values are given in the second row of **Table 1.4**.

Results of the calculation of the efficiency e_G of the vessel coordinates in the case of the distribution of position line errors according to the mixed law of the second type with density $g_2(x)$, and the calculation of the coordinates was performed by the least squares method given in [14]. The efficiency e_G values are also given in **Table 1.4**.

Table 1.4 Efficiency e_G and e_R distribution density $g_1(x)$

n	**3**	**4**	**5**	**6**	**8**	**10**
e_G	0.917	0.945	0.962	0.971	0.982	0.988
e_R	0.996	1	1	1	1	1

Analyzing **Table 1.4**, let's note the high efficiency e_R of the ship's coordinates calculated by the proposed method of applying orthogonal decomposition, which exceeds the efficiency e_G coordinates obtained by the least squares method.

Thus, by applying the orthogonal expansion of the density of errors of the position lines and calculating the coordinates of the vessel's location using the system of equations (1.3), the dispersion of the observation error is reduced, which leads to increased energy efficiency.

1.5 Conclusion

In conclusion, improving the energy efficiency of ships is an important step in reducing the shipping industry's impact on the environment. There are various strategies and technologies that can be employed to achieve this goal, including hull coatings, waste heat recovery, energy management systems, hybrid propulsion systems, and wind propulsion. While each strategy has its advantages and limitations, their combined use can help improve the overall energy efficiency of ships and reduce their impact on the environment.

Study demonstrates that significant improvements in vessel energy efficiency can be achieved through enhanced navigation methods. Key conclusions include:

1. The potential for fuel consumption reduction through combined optimization approaches.
2. The feasibility of implementing proposed methods without major technical modifications.
3. The importance of integrating multiple optimization strategies for maximum effectiveness.

The findings support the development of more sophisticated navigation protocols that prioritize energy efficiency while maintaining operational safety and effectiveness. The study contributes to the broader field of maritime energy efficiency by providing practical, implementable solutions based on navigation optimization.

The proposed method of determining the coordinates of the vessel using orthogonal expansion of the density of the error distribution provides higher efficiency compared to the least square method. At the same time, for both types of mixed error distribution laws, almost full efficiency (close to 1) is achieved already at $n \geq 4$. Reducing the dispersion of the observation error allows to increase the accuracy of determining the location of the vessel and, as a result, improve its energy efficiency.

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