CHAPTER 1

The concept of transition to a balanced sustainable development model

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Abstract

Sustainable Development Goals pose new challenges for economies, which need to adapt their operations and strategies to the requirements of the SDGs. The object of the study is the system of ecological, economic, social and political relations in the process of sustainable development of a market economy in the context of implementation of the provisions of the Paris Agreement. The issue to be solved is analyzing the concept of sustainable development, the methodology for studying the interaction between the economy and the environment, to clarify their content using a balanced approach to economic, social and environmental tasks in modern conditions, and, as a result, to identify the main trajectories among the possible ones. The mathematical tools for modeling and analyzing the sustainable development of the economic system with regard to the Paris Agreement have been developed; the proposed methodology and tools allow analyzing the impact of environmental constraints on economic development at the level of intersectoral interaction. The interpretation of the results obtained is that it is possible to determine the macroeconomic parameters of the national sustainable development with ensuring both economic and environmental balance. The practical experience of using inter-sectoral models to solve environmental problems and rational use of resources shows the need for their application in the system of macroeconomic sectoral and regional models, and the obtained main trajectories – in the system of long-term planning. The peculiarity of the obtained results lies in the application of the method of basic matrices to the study of balance ecological and economic models of "input-output" based on the method of basic matrices and the possibility of its application in conditions of poor conditionality of the matrix structure. The proposed provisions,

recommendations and conclusions can be used in the sustainable development of relevant legislation and laws at the national and international levels.

Keywords

Sustainable development goals, Paris Agreement, matrix ecological and economic models, productivity, parametric system synthesis.

1.1 Introduction

The modern type of economic development is defined as a technogenic type of socio-economic development. This type can be characterized as a nature-intensive type of sustainable development based on the use of artificial means of production that are created and function without regard to environmental constraints and stimulate the unlimited growth in the consumption of goods. As a result of human activity, the environment has begun to have an unprecedentedly destructive impact. If the current trends continue, the use of natural resources, pollution, and greenhouse gas emissions will increase several times over the next half century. Meanwhile, sustainable economic development must take into account at least three increasingly obvious environmental constraints:

– limited capacity of the environment to absorb, assimilate various kinds of waste, pollution, greenhouse gases caused by economic systems;

– degradation of renewable natural resources as a result of overexploitation;

– a finite amount of non-renewable resources.

Ignoring these limitations and the unlimited development of the technogenic type of the world economy has led to global environmental problems. As defined in the third assessment report of the Intergovernmental Panel on Climate Change, "climate change is a problem with unique characteristics. It is global, long-term and encompasses a complex of interactions between climatic, environmental, economic, political, institutional, social and technological processes" [1]. Thus, unlike other global problems, global climate change is more complex, multisystem, multilevel and multicomponent.

The adoption by the United Nations of the Paris Agreement (PA) on climate protection, which, in particular, aims to reduce greenhouse gas emissions globally, was a relevant step in preventing further deepening of these threats and the problems of the negative impact of climate change on sustainable socio-economic development [2].

In early July 2021, the European Commission presented a package of legislative initiatives "Fit for 55" to implement the European Green Deal and ensure

a 55% reduction in greenhouse gas emissions in the EU by 2030 from 1990 levels. In October 2023, the work on preparing the necessary legislative framework was finally completed [3]. Relevant initiatives were also adopted by the Government of Ukraine in the form of the Updated National Determined Contribution to the Paris Agreement (NDC2), which declares a national target for reducing greenhouse gas emissions. By 2030, this reduction should be 65% compared to 1990 [4].

An illustrative instance may be seen in the study conducted by W. Zhang, M. Zhang, Sh. Wu, F. Liu, whereby the authors employed the four indicators of low-carbon economic benefits, low-carbon technology stock, low-carbon human resources, and CO_2 emissions reflect the level of low-carbon sustainable development of enterprises. The results show that the level of low-carbon sustainable development of enterprises is not high (<0.6); low-carbon human resources have two stages of decline and rise; government supervision and low-carbon technology can significantly reduce CO_2 emissions. However, multi-scenario simulation results show that in the early stage of low-carbon sustainable development of enterprise, excessive government supervision will reduce the low-carbon economic benefits of enterprise. The scholars indicate that within 10 years, policy supervision can reduce CO_2 emissions by up to 67% and increase low-carbon benefits by 17%; low-carbon technology can reduce CO_2 emissions by up to 60% and increase low-carbon benefits 33% [5].

It is increasingly clear that the traditional model of the economic growth, which ignores the importance of natural factors and is unable to prevent the aggravation of global environmental problems, has exhausted itself. Thus, the development of new predictive models of the sustainable development with a feedback loop – a mechanism for analyzing the effects of the implementation of sustainable development guidelines on the participants in the process, including the environment – has become urgent.

In this case, it would be a reasonable step to move to the level of mathematical modeling of ecological and economic interaction as an effective tool for scientific knowledge. Including the construction of the sustainable development highway in the context of the study of the ecological and economic process in the course of modeling provides a feedback in the system. So, the solution of the main problem mathematically determines the relevant structural elements of the system (matrix and vector of constraints), and also indicates how the found (desired) parameters of reorganization correspond to the inherent technological capabilities (tolerances) for changes. The direction and size of the sustainable development step along the main line should also be in line with the permissible changes. The study of the aforementioned inverse problem to realize the directed evolution of the system in the course of modeling is important and relevant.

1.2 Key concept of mathematical modeling in ecology and economy

The issue of mathematical modeling of the interaction between the economy and the environment within the framework of the Kyoto Protocol is not new. The first class of such models can be called inter-sectoral balance models, which investigate the impact of the economic structure on the environment. According to L. Buiak, O. Bashutska, K. Pryshliak, V. Hryhorkiv, M. Hryhorkiv and V. Kobets, this class of models includes the Leontief-Ford inter-sectoral model and its generalization. In our opinion, the main disadvantage of this class of models is their inability to represent the impact of economic instruments for reducing greenhouse gas emissions on the behavior of market participants. At the same time, a fundamental issue from an economic point of view is related to the extent to which the system formed by the coal market allows economic agents to fulfill their obligations at minimal cost [6].

Numerous studies show that such entities will receive significant economic benefits from participating in the agreement, taking into account the cost of the carbon allowance, as well as the associated benefits from greenhouse gas emission reduction projects. At the same time, C. Böhringer, S. Peterson, T. Rutherford, J. Schneider, M. Winkler suppose that the Kyoto Protocol and the Paris Agreement may create barriers to economic growth if greenhouse gas emissions exceed the level of obligations set for the agent. Thus, there is a need to study a complex, integrated economic and environmental problem and find a balanced solution [7].

Furthermore, the models proposed by R. Sun and studied for the construction of main trajectories in the case of dynamic systems do not reflect their analogues for static balance models. This fact causes a gap and lack of full interconnection between static and dynamic input-output balance models. Moreover, he supposes that economic models play a crucial role in policy formulation, enabling proactive analysis to minimize unintended consequences and maximize policy effectiveness, benefiting both individuals and society. These models also aid decision-making by providing quantitative frameworks to evaluate different options, reducing uncertainty and risk [8].

The analysis of balance ecological-economic models by V. Hryhorkiv, L. Buiak, M. Hryhorkiv is based on the study of productivity and does not reflect the impact of controlled changes in technological matrices on the vectors of gross output of the main and auxiliary ecological production. Despite the diversity of such problems, a single functional-analytical approach can be used to study them. Methods for solving systems of linear algebraic equations (SLAE) are fundamental in a number of problems (and not only environmental and economic ones), since it is to their study that the analysis and optimization in the initial formulation is reduced (after simplification and discretization) [9]. High-precision computational schemes in rational

and long mantissa numbers are proposed by V. Kudin, V. Onotskyi, A. Al-Ammouri, L. Shkvarchuk. The scholars sustain that comparative schemes of algorithms on the corresponding test libraries were logically supplemented by their application to applied matrix models [10].

At the same time, in contrast to the matrix structures identify that it is logical to consider more general variants of matrix models, for instance, in conditions of poor determinacy of the matrix structure. This will deepen the methodology for modeling and creating a set of mathematical methods and models for the development of the ecological and economic system in the context of implementing a policy to reduce greenhouse gas emissions, the variables of which include the main sustainable economic indicators and the volume of emission quotas as an environmental resource. In our opinion, this should complement the existing methods of management decision-making and increase the efficiency of the economy's transition to sustainable development [10].

Given the current level of production development and the existing production structure, the primary importance in solving the problem of balance is to determine the right priorities in the sustainable development of industries and regions aimed at achieving high end results and accelerating the greening of the sustainable economy. Establishing and maintaining priorities and proportions requires determining the resources required for their implementation, which to some extent complicates the achievement of balance. In such circumstances, it is important to prevent all other sectors from lagging behind and postponing the solution of non-priority but important problems. Prioritized sustainable development of certain industries and intensification of industrial production imply certain structural shifts in the economic system and require appropriate proportions in the growth rates of various structural components of the sustainable economy. The process of optimal system management aimed at flexible response to changes in the volume and structure of needs and the corresponding impact on the volume and structure of production has an important impact on structural changes. Such management actively contributes to improve the structure of the sustainable economy and stabilizing greenhouse gas emissions in current and future planning.

These criteria meet the conditions of the main trajectory, which generally characterizes the optimal program of ecological and economic growth. Furthermore, one of the main results of the analysis of balance-type models is the proof of the existence of a backbone characterized by an unchanged production structure. However, it is not always possible for the system to develop along the main line, i.e. the main line is not always an acceptable trajectory for a given initial state. Nevertheless, the normative value of the backbone is quite important.

The following result follows from the ecological and economic analysis of balance models. The overall planning horizon of the ecological and economic system consists of three segments, namely: movement to a stationary point, movement in the mode of balanced development and transition to the end point. At the same time, the trajectory of balanced exponential growth or the highway as a whole characterizes the optimal program of ecological and economic growth the better, the longer the period of balanced development.

The issue of choosing the optimal development trajectory for a given initial and final state was developed as a basic principle. This principle is based on the following hypothesis: effective trajectories at intermediate stages before the transition to the final state tend to follow the Neumann ray and the longer the horizon of planning the sustainable development of the economic system, the more the effective trajectory coincides with this ray. Samuelson's principle is a strategy for effective long-term economic growth. It means that a system that has left its initial state should reach the ray of maximum balanced growth and then to function for as many planned periods as possible in the mode of such growth, and then to reach the final state.

The validity of the above principle is proved in the mainline theorems. These theorems state that for a sufficiently long period of time, the optimal trajectory of a dynamic model, regardless of the initial and final states or the objective function, is close to the backbone. Models and trajectories for which such theorems are valid are called backbone models. In our opinion, from the backbone theorems, it is possible to formulate the following conclusion: the duration of the start and end segments in the trajectory does not depend on the length of the planning period. Accordingly, as the planning period increases, the part of the trajectory that is located on the main line or in its vicinity increases. The effective trajectory for a long planning period depends mainly on the structural parameters of the model and to a lesser extent on the initial and final states.

Samuelson's principle makes it possible to simplify and solve the problem of choosing an objective function. In particular, the task of choosing end conditions in dynamic problems of prospective sustainable development, to simplify the calculation and analysis of optimal trajectories of the economic system. Since most of these trajectories are close to or coincide with the trajectories of maximum growth. Let's consider that the latter implies the availability of a mathematical apparatus for taking into account the impact of changes (refinements) on the properties of the new model, of course, without the procedure of solving the problem again (initially). It should be noted that models of ecological and economic processes (as linear systems) have a block (cellular) structure – quadrants of the constraint

matrix. In particular, the classical scheme of inter-sectoral balance in the first quadrant contains inter-sectoral flows that correspond to functional and structural sectoral relations.

V. Kudin, A. Onyshchenko, I. Onyshchenko affirmed that the relevance of taking this property into account in the modeling process determines the expansion of the scope of application of methods and algorithms to analyze group changes in blocks (quadrants) of the model matrix on the properties of a linear system. In particular, solutions are in the course of improving the model of environmental and economic processes [11].

Thus, it becomes important to introduce a static model adapted to the nature of the development highway and to build appropriate mathematical algorithms. They should be applied to balance ecological and economic models in order to develop ways to transfer the system from one state to another. That is set according to economic and environmental criteria in the study of inverse and trajectory problems. For this purpose, it is necessary to analyze the use of the balance ecological and economic models of the "input-output" type, which provide the allocation of a part of the national product for the implementation of measures to fulfill obligations under the Paris Agreement in order to study the sectoral structure of the economy.

In our opinion, it is necessary to investigate the issue of a targeted transformation of the sectoral structure of the economy using input-output balance ecological and economic models, to build the corresponding main trajectories based on the matrix theory of pseudo-reversal in the context of the implementation of the sectoral program to limit greenhouse gas emissions. The technology of the method of basis matrices is based on solving (and re-solving) direct problems in particular, by the impact of changes on the properties of the model without solving the problem from the beginning. Establishing links between direct and inverse problems based on the method of basis matrices makes it possible to study inverse, mainstream problems (implementation of sustainable development) and also to apply computational procedures (post-solutions) developed for direct problems.

1.3 Sustainability performance

The change in ecosystem services values under plausible scenarios highlight the fundamental role that healthy ecosystems have in achieving global objectives such as the Aichi Targets [12], especially targets 5, 14, 15 of the Convention of Biological Diversity (**Fig. 1.1**), the Sustainable Development Goals [13], in which natural capital (SDGs 6, 7, 13, 14 and 15) is the cornerstone that sustains the rest of the

goals, and the 1.5°C temperature increase limit of Paris Agreement of the United Nations Framework Convention on Climate Change [14]. O. Lytvyn debates that the governments should consider incorporating SDG principles into their corporate governance policies and follow relevant guidelines when planning and designing projects [15].

SDGs	Aichi Biodiversity Targets
6. Ensure the availability and sustainable management of water and sanitation for all	8, 11, 14, 15
7. Ensure access to affordable, reliable, sustainable and modern energy for all	5, 7, 14, 15, 19
13. Take urgent action to combat climate change and its impact	2, 5, 10, 14, 15, 17
14. Conserve and sustainably use the oceans, seas and marine resources for sustainable development	2, 3, 4, 5, 6, 7, 8, 10, 11, 12, 14, 15, 17, 19
15. Protect, restore and promote sustainable use of terrestrial ecosystems, sustainably manage forests, combat desertification and halt and reverse land degradation and halt biodiversity loss	2, 4, 5, 7, 9, 11, 12, 14, 15, 16
Overview of Aichi Biodiversity Targets	

Fig. 1.1 Correlation among SDGs and Aichi Biodiversity Targets in 2024
Source: compiled by the authors based on [12, 13]

The cumulative impact of multiple environmental crises is threatening the foundations of planetary ecosystems. In 2023, the world experienced the warmest year on record. For the first time, global temperatures were dangerously close to the 1.5°C lower limit of the Paris Agreement. Global greenhouse gas emissions and atmospheric concentrations of carbon dioxide reached new records yet again in 2022, with no signs of slowing in 2023. Developing and vulnerable countries face vast development challenges. Per capita growth in gross domestic product (GDP) in half the world's most vulnerable countries is now slower than in advanced economies for the first time this century [16]. This trajectory threatens to reverse a long-term

trend towards more income equality among countries. Furthermore, after a decade of rapid debt accumulation, the external debt stock in low- and middle-income countries remains at unprecedentedly high levels.

The progress assessment carried out in 2024 reveals that the world is severely off track to realize the 2030 Agenda. Of the 169 targets, 135 can be assessed using available global trend data from the 2015 baseline to the most recent year, along with custodian agency analyses; 34 targets lack sufficient trend data or additional analysis [16]. Among the assessable targets, only 17 percent display progress sufficient for achievement by 2030 (**Fig. 1.2**).
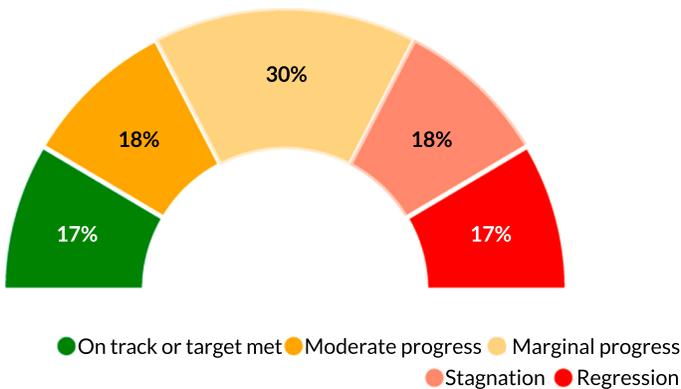

Fig. 1.2 Overall progress across targets based on 2015–2024 global aggregated data
Source: [16]

Nearly half (48%) exhibit moderate to severe deviations from the desired trajectory, with 30% showing marginal progress and 18% moderate progress. Alarmingly, 18% indicate stagnation and 17% regression below the 2015 baseline levels (**Fig. 1.3**). This comprehensive assessment underscores the urgent need for intensified efforts to put the SDGs on course.

As part of sustainable development, O. Lytvyn, A. Onyshchenko and O. Ostapenko sustain that post-war companies in Ukraine should increase their activities in the field of renewable energy and ecology, as well as support environmental projects in any way possible, thus contributing to the achievement of the 6th, 7th and 13th Global Goals for Sustainable Development, which the United Nations has set for 2015. Ukrainian companies should be aware of their environmental impact and strive to reduce it by developing their sustainability orientation, including sustainable business and sustainable management [17].
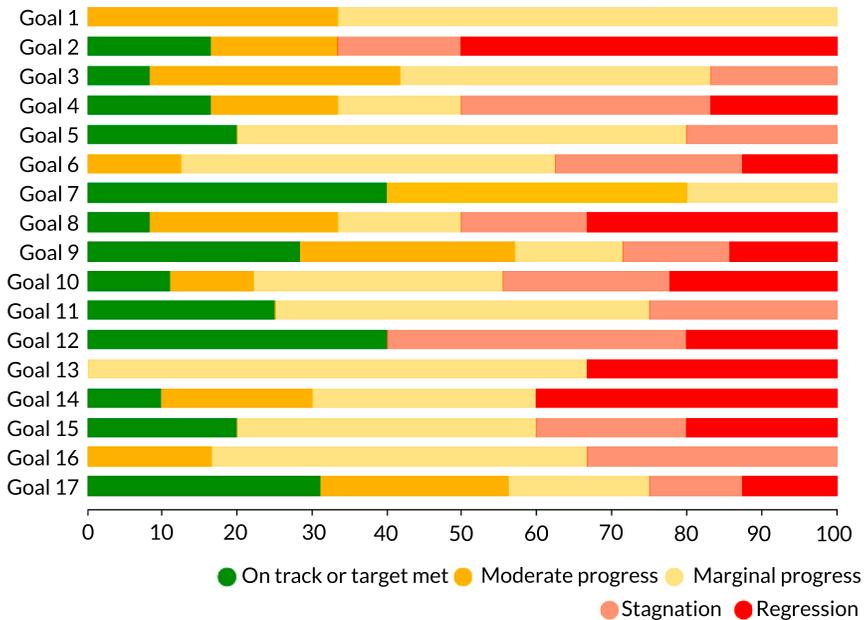
Fig. 1.3 Progress assessment for the 17 Goals based on assessed targets in 2024, %
Source: [16]

By 2030, one of the main SDG targets – SDG 6 clean water and sanitation is to achieve universal and equitable access to safe and affordable drinking water for all and to adequate and equitable sanitation, and hygiene for all and end open defecation [12, 16]. According to **Fig. 1.4**, only target 6.2 – sanitation and hygiene has a moderate progress in 2024 but sanitation needed. By 2030, one of the main SDG targets – SDG 7 affordable and clean energy is to ensure universal access to affordable, reliable and modern energy services and increase substantially the share of renewable energy in the global energy mix [12, 16]. According to **Fig. 1.4**, only target 7.a – international cooperation on energy and target 7.b – investing in energy infrastructure are on track in 2024.

By 2030, one of the main SDG targets – SDG 13 climate action is to strengthen resilience and adaptive capacity to climate-related hazards and natural disasters in all countries and Integrate climate change measures into national policies, strategies and planning [13, 16]. According to **Fig. 1.4**, only target 13.1 – resilience and adaptive capacities has a marginal progress in 2024 and a significant acceleration needed.

6 CLEAN WATER AND SANITATION

6.1 Safe drinking water ○
6.2 Access to sanitation & hygiene ●
6.3 Water quality ○
6.4 Water-use efficiency ○
6.5 Trans-boundary water cooperation ○
6.6 Water-related ecosystems ○
6.a International cooperation on water & sanitation ●
6.b Participatory water & sanitation management ○

7 AFFORDABLE AND CLEAN ENERGY

7.1 Access to energy services ●
7.2 Share of renewable energy ●
7.3 Energy efficiency ○
7.a International cooperation on energy ●
7.b Investing in energy infrastructure ●

13 CLIMATE ACTION

13.1 Resilience & adaptive capacity ○
13.2 Climate change policies ●
13.3 Climate change awareness ●
13.a UNFCCC commitments ○
13.b Climate change planning & management ●

14 LIFE BELOW WATER

14.1 Marine pollution ○
14.2 Marine & coastal ecosystems ○
14.3 Ocean acidification ●
14.4 Sustainable fishing ●
14.5 Conservation of coastal & marine areas ○
14.6 Fisheries subsidies ●
14.7 Marine resources for SIDS & LDCs ●
14.a Research capacity & marine technology ●
14.b Small-scale artisanal fishing ●
14.c Implementing UNCLOS ●

15 LIFE ON LAND

15.1 Terrestrial & freshwater ecosystems ○
15.2 Sustainable forests management ○
15.3 Desertification and land degradation ●
15.4 Conservation of mountain ecosystems ○
15.5 Loss of biodiversity ●
15.6 Utilization of genetic resources ●
15.7 Protected species trafficking ●
15.8 Invasive alien species ●
15.9 Biodiversity in national & local planning ○
15.a Resources for biodiversity & ecosystems ○
15.b Resources for forest management ○
15.c Protected species trafficking (global) ●

● On track or target met   ○ Stagnation
● Moderate progress, but acceleration needed   ● Regression
○ Marginal progress, and significant acceleration needed   ● Insufficient data
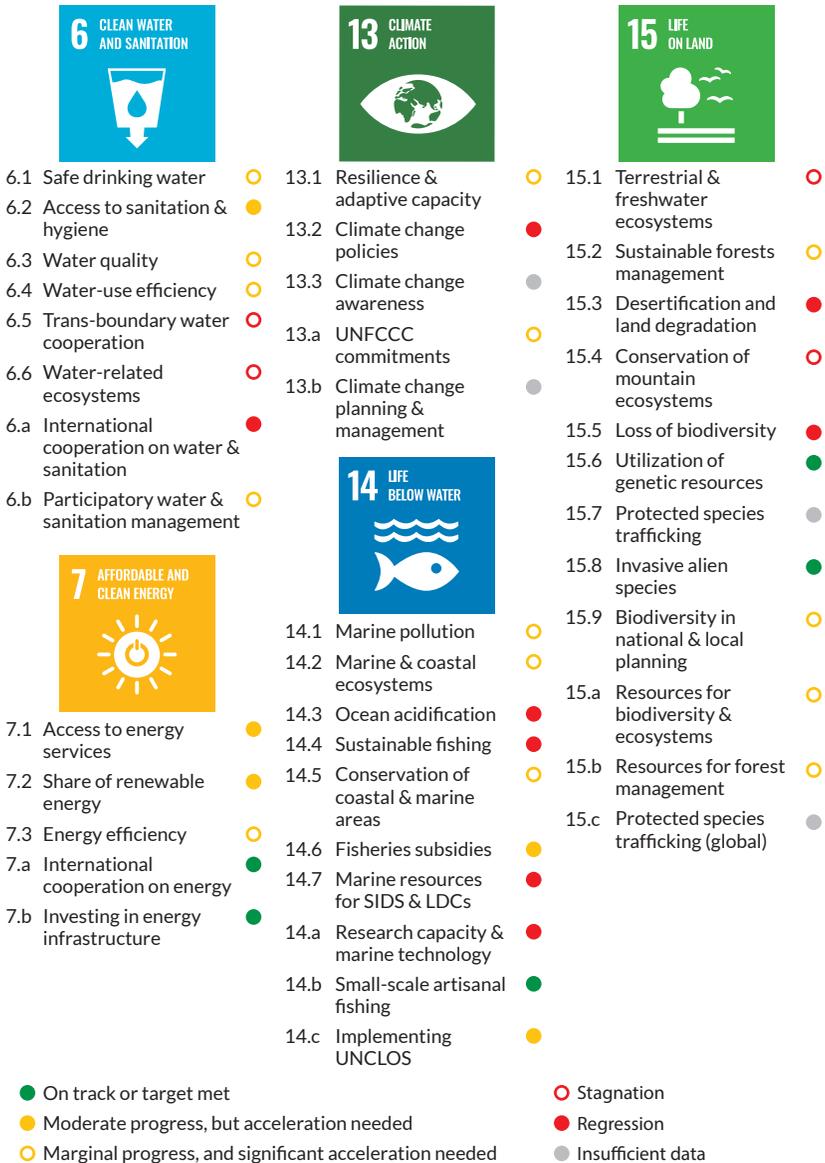
Fig. 1.4 SDG progress by target in 2024
Source: compiled by the authors based on [13, 16]

By 2025, one of the main SDG targets – SDG 14 life below water is to prevent and reduce marine pollution of all kinds, in particular from land-based activities, including marine debris and nutrient pollution [13, 16]. According to **Fig. 1.4**, only target 14.b – small-scale artisanal fishing is on track in 2024. By 2030, one of the main SDG targets – life on land is to ensure the conservation of mountain ecosystems, including their biodiversity, in order to enhance their capacity to provide benefits that are essential for sustainable development [13, 16]. According to **Fig. 1.4**, only target 15.6 – utilization of genetic resources and target 15.7 – invasive alien species are on track in 2024.

In recent years, the delicate balance between economic development and ecological environment protection in ecologically fragile arid areas has gradually become apparent. J. Liu, X. Pei, W. Zhu, J. Jiao indicate that deep learning models effectively captured the spatiotemporal neighborhood features of land use dynamics, and CNN-GRU exhibited the highest accuracy and most accurately simulated the land use [18]. C. Paul, E. Reith, J. Salecker outline how future hybrid models could build on recent advances by inter alia an improved representation of landscape patterns, refining the theory behind decision-making, incorporating uncertainty and reducing model complexity. The scholars conclude that coupling recent developments in land-use optimization and agent-based models may help bridge gaps between modelling philosophies as well as parsimony *vs.* complexity [19].

In the sustainable use area, clarifying the complex land use impacts on ecosystem services, X. Cheng, Z. Xu, Sh. Yu, J. Peng consider that trade-off will be beneficial to watershed sustainable development, especially through scientific land use management and decision making [20]. The trade-off intensity is the strongest in farmland and forest land, and weaker in grassland and water body. Bioeconomic modelling has been positioned as an integrative approach able to deliver advice to sustainability related problems. L. Castro, F. Lechthaler see a tremendous evolution towards integrated based on multiple-goal and spatial applications. This step forward in bioeconomic modelling enables it to incorporate multiple time and spatial scales [21].

In a broader context, C. King derives a long-term dynamic growth model that endogenously links biophysical and sustainable economic variables in a stock-flow consistent manner. The two industrial sector HARMONEY (Human And Resources with MONEY) model enables exploration of interdependencies among resource extraction rate and depletion; the accumulation of population, capital, and debt; and the distribution of money flows within the sustainable economic development [22].

According O. Lytvyn, Y. Kuryliuk, A. Onyshchenko, V. Kudin, V. Parkhomenko and S. Filiuk, this once again proves that the path of sustainable development is inevitable and the concept of social responsibility should reflect rationality. Conflicting

expectations of the entire set of interested parties, should be based on the principles of continuous and long-term development of business entities, with the aim of obtaining competitive advantages [23].

The complexity of the socioeconomic evolution circumscribed by the accumulation of irreversible irregularities, dissipative structures, and is a challenge with an impact on the decision-making process. In this context, sustainable development involves homeoresis and resilience under economic, ecological, and environmental constraints. Mathematical modeling in the analysis of economic, ecological, and environmental phenomena, configured in various systemic structures, allows the consideration of multiple variables, identification of conditionalities, facilitating the adoption of appropriate decisions. On these coordinates, mathematical modeling offers multiple possibilities of analysis, constituting a useful tool at the micro- and macroeconomic level.

1.4 Input-output balanced sustainable development model with consideration of environmental management processes

The aim of the study is also to develop an algorithm for synthesizing the components of a static ecological and economic mathematical model of the Leontief-Ford type of input-output (in particular, the constraint matrix and the constraint vector), the solution of which implements a given sustainable development highway.

The main issue that arises while creating a long-term plan for environmental protection and rational use of natural resources at the macroeconomic level is to justify the required amount of resources and their distribution among individual industries and regions of the country. Let's consider a model that allows to determine macroeconomic parameters of the national economy development that ensure not only economic but also environmental sustainability (**Table 1.1**). It presents a general scheme of the extended input-output one, which reflects not only flows between the sectors of conventional goods and services, but also the production and destruction of pollutants. Accordingly, the usual classification of economic activities and goods is expanded to include the names of various pollutants and measures to eliminate them.

The construction and implementation of the input-output balanced sustainable development model, based on the scheme presented in the **Fig. 1.5** allows solving a number of fundamental problems of the modern science. These problems include, in particular: development of reliable methods for predicting environmental parameters and indicators of its quality, which provide quantitative measurement of

human satisfaction in a clean environment; creation of a scientifically based methodology for determining economic losses from environmental pollution; construction of a system of models of interaction between different components of natural complexes, taking into account anthropogenic factors.

Table 1.1 Input-output balanced sustainable development model with consideration of environmental management processes

PRODUCTION	Sustainable Development Goals: 6, 7, 13, 14, 15						
	Consumption						
	SOCIETY			NATURE			
	Manu-facturing industries	Non-manu-facturing industries	Natural resource industries	Biota	Atmo-sphere	Hydro-sphere	Litho-sphere
	1	2	3	4	5	6	7
SOCIETY							
1 Manufacturing industries	*I quadrant* **Economic relations** Aichi Biodiversity Targets: 3, 4, 6, 7			*II quadrant* **Environmental impact** (pollution, radiation, vibration, noise and other types of anthropogenic impact) Aichi Biodiversity Targets: 5, 6, 8, 11, 14, 15			
2 Non-manufac-turing industries							
3 Natural resource industries							
NATURE							
4 Atmosphere	*III quadrant* **Use of natural resources** Aichi Biodiversity Targets: 13, 16, 20			*IV quadrant* **Environmental connections** Aichi Biodiversity Targets: 1, 2, 4, 10, 12, 17, 18, 19			
5 Atmosphere							
6 Hydrosphere							
7 Lithosphere							

Source: compiled by the authors

From a scientific point of view, it requires theoretical research to establish the conditions for the existence of a corresponding mathematical model for a given development path, in particular, with positive elements and a solution that implements it. In general, the purpose is to expand the range of research to include the solution of inverse problems.

At the same time, in accordance with the purpose, the task is also to set in the practical plane and establish the necessary (sufficient) conditions for the existence (non-existence) of a mathematical model (constraint matrix, constraint vector and variables) under given tolerances (restrictions on parameter values),

the solution of which implements a given development path. The task is also to develop a procedure for evaluating organizational innovations due to the selected sustainable development priorities (according to the main line). Quantitative changes can occur in both the matrix and the constraint vector of the model.

The following objectives were set to achieve the aim:

– based on the analysis, to formulate an analog of the sustainable development highway for a static model (problem) and select basic methods and algorithms for solving it;

– develop new and improve existing approaches (based on basic methods and algorithms) to solving the problem;

– conduct a computational experiment to test the properties of the algorithms for solving the problem.

The object of the study is mathematical models of static ecological and economic processes.

The subject of the study is methods and algorithms for analyzing and solving linear systems.

1.5 Description of ecological and economic models balance in the implementation of major changes

L. Buiak, O. Bashutska, K. Pryshliak, V. Hryhorkiv, M. Hryhorkiv and V. Kobets noted that Leontief-type balance ecological and economic models play an important role in solving environmental problems. Within the framework of such models, it is possible optimally to combine groups of production and environmental industries, to consider their interconnections and interaction. At the same time, it is proposed to take into account the costs of meeting greenhouse gas emission limits in the structure of the main production sectors [6]. Structurally, ecological and economic models presented by V. Kudin, A. Onyshchenko, I. Onyshchenko can be meaningfully interpreted as cellular (block). Certain interconnections can be traced between the blocks in this matrix [11].

The balance of the economy as a form of systematic interconnection of production and consumption implies that the volume and structure of produced products and services corresponds to the volume and structure of social needs in natural and value terms. That is, in time and space, in terms of quality and quantity. This correspondence is achieved by introducing a priority MDP (making decision person) vector (highway), according to the importance of the economic and environmental components of the system's output.

V. Kudin, A. Onyshchenko, I. Onyshchenko proposed to take into account the costs of meeting greenhouse gas emission limits in the structure of the main production sectors in the form of a balance ecological and economic model:

$$\begin{cases} x_1 = A_{11}x_1 + A_{12}x_2 + Cy_2 + y_1, \\ x_2 = A_{21}x_1 + A_{22}x_2 - y_2, \end{cases} \tag{1.1}$$

where $x_1 = (x_1^1, x_2^1, \ldots, x_n^1)^T$ – a vector-column of production volumes; $x_2 = (x_1^2, x_2^2, \ldots, x_m^2)^T$ – a vector-column of destroyed pollutants' volumes; $y_1 = (y_1^1, y_2^1, \ldots, y_n^1)^T$ – a vector-column of final products' volumes; $y_2 = (y_1^2, y_2^2, \ldots, y_m^2)^T$ – a vector-column of volumes of undestroyed pollutants; $A_{11} = (a_{ij}^{11})_1^n$ – a square matrix of coefficients of direct costs of product i for the production of a unit of product j; $A_{12} = (a_{ig}^{12})_{i,g=1}^{n,m}$ – a rectangular matrix of product i costs per unit of pollutant destruction g; $A_{21} = (a_{kj}^{21})_{k,j=1}^{m,n}$ – a rectangular matrix of pollutant output k per unit of manufactured product j; $A_{22} = (a_{kg}^{22})_1^m$ – a square matrix of pollutant output k per unit of pollutant destruction g; $C = (c_{ig}^{12})_{i,g=1}^{n,m}$ – a rectangular matrix of product i consumption per unit of pollutant g emissions.

In system (1.1), it is implicitly assumed that the coefficients $a_{ij}^{11} \geq 0$, $a_{ig}^{12} \geq 0$, $a_{kj}^{21} \geq 0$, $a_{kg}^{22} \geq 0$ apply to all types of production activities (material production and pollutant elimination) the hypotheses of the basic model of inter-industry balance: the number of technological methods equals the number of types of products and each technological method produces only one type of product. In the following, it is possible to assume that matrices $A_{11}, A_{12}, A_{21}, A_{22}$ are nonnegative: $A_{11} \geq 0, A_{12} \geq 0, A_{21} \geq 0, A_{22} \geq 0$. The economic sense of the model (1.1) requires that all its variables are nonnegative, i.e. $x_i^1 \geq 0, x_k^2 \geq 0, y_i^1 \geq 0, y_k^2 \geq 0$.

The latter is closely related to the issue of the performance of balance models, which allows to talk about the real functioning of the production system capable of ensuring intermediate consumption, positive volumes of the final product and compliance with the established limits on greenhouse gas emissions. Achievement of their main values, i.e., the specified variables (output indicators), causes structural reorganization (changes) in individual elements, rows, columns of technological matrices of the model (1.1), as well as in the forming cells (blocks). This necessitates the development of algorithms considered by P. Samuelson for assessing the impact of changes in the blocks of the matrix structure on the solution of the system of equations [24].

The first set of equations of the proposed model reflects the economic balance – the distribution of sectoral gross output into the 'production consumption' of a primary and secondary production; a final consumption of the primary production,

and costs associated with fulfilling obligations under the Paris Agreement [14]. The second set of equations reflects the physical balance of greenhouse gases as the sum of emissions caused by the activities of the main and auxiliary production and their unremoved volumes.

According to [6, 24], let's specify the model in the form

$$Au = \widetilde{C}, \tag{1.2}$$

where $A = \begin{pmatrix} E_1 - A_{11} & -A_{12} \\ -A_{21} & E_2 - A_{22} \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \end{pmatrix}$, $u = (x_1, x_2)^T = (u_1, u_2, ..., u_m)^T$ – an m-dimensional vector; x_1, x_2 – "subvectors" u; $\widetilde{C} = \begin{pmatrix} E_1 & C \\ 0 & -E_2 \end{pmatrix} \begin{pmatrix} y_1 \\ y_2 \end{pmatrix}$; E_1, E_2 – block unit matrices of the corresponding dimension; 0 – block zero matrix; Cy_2 – costs of greenhouse gas emissions (i.e., the cost of servicing greenhouse gas emissions, in particular, the cost of emission permits); $C = (c_{ig}^{12})_{i,g=1}^{n,m}$ – a rectangular matrix of product i costs per unit of pollutant g emissions.

Other variables of the system are similar in content to the corresponding variables of the model [6].

Let's also consider the system

$$\overline{A}u = \overline{C}, \tag{1.3}$$

reorganized in the way of achieving the main growth indicators (in the elements of matrices A_{11}, A_{12}, A_{21}, A_{22} and C) with respect to the system of linear algebraic equations (1.2).

In general, let's believe that the constraint vector and the constraint matrix should not "fall out" of a certain parallelepiped of variable technological constraints with their permissible value:

$$\Pi_{\overline{A}} = \left\{ \overline{A} / \overline{A}_{(H)} \leq \overline{A} \leq \overline{A}_{(B)} \right\}, \Pi_{\overline{C}} = \left\{ \overline{C} / \overline{C}_{(H)} \leq \overline{C} \leq \overline{C}_{(B)} \right\}, \Pi_u = \left\{ u / u_{(H)} \leq u \leq u_{(B)} \right\}.$$

A more detailed record will be used further:

$$\Pi_{\overline{a}_i} = \left\{ \overline{a}_i / a_{i(H)} \leq \overline{a}_i \leq a_{i_{(B)}}, i \in I \right\}, \Pi_{\overline{c}_i} = \left\{ \overline{c}_i / c_{i(H)} \leq \overline{c}_i \leq c_{i_{(B)}}, i \in I \right\}, \Pi_{u_i} = \left\{ u_i / u_{i(H)} \leq u_i \leq u_{i_{(B)}}, i \in I \right\}.$$

It should be noted that such a problem of parametric synthesis to find the structure of the system (constraint matrix and constraint vector). It is possible to assume that the solution of such a problem corresponds to the development of the main line

if the solution of the problem (1.3) on the synthesized model structure coincides or is close to the given main line.

Accordingly, the statement can be as follows:

Let's form the initial model as an initial, unperturbed model of type (1.2):

$$\max Bu \,/\, Au = C,$$

$$\Pi_u = u \,/\, \overline{u}_{(H)} \leq u \leq \overline{u}_{(B)}.$$

It is possible to investigate the optimality of the parameters (all model parameters are unknown and bilaterally constrained) for a perturbed problem of type (1.3):

$$\max \overline{B}u \,/\, \overline{A}u = \overline{C},$$

$$\Pi_{\overline{A}} = \left\{ \overline{A} \,/\, \overline{\overline{A}}_{(H)} \leq \overline{A} \leq \overline{\overline{A}}_{(B)} \right\},$$

$$\left(\Pi_{\overline{a}} = \{ \overline{a}_1 \,/\, \overline{\overline{a}}_{i(H)} \leq \overline{a}_i \leq \overline{\overline{a}}_{i(B)} \}, i = \overline{1,m} \right),$$

$$\Pi_{\overline{C}} = \left\{ \overline{C} \,/\, \overline{\overline{C}}_{(H)} \leq \overline{C} \leq \overline{\overline{C}}_{(B)} \right\},$$

$$\Pi_{\overline{B}} = \left\{ \overline{B} \,/\, \overline{\overline{B}}_{(H)} \leq \overline{B} \leq \overline{\overline{B}}_{(B)} \right\},$$

$$\Pi_u = \left\{ u \,/\, \overline{u}_{(H)} \leq u \leq \overline{u}_{(B)} \right\}.$$

By applying, for instance, the penalty function method, problem (1.3) of conditional optimization is reduced to the problem of unconditional optimization of the function for a minimum under bilateral constraints on the parameters:

$$\min \left\{ -\overline{B}u + \sum_{j=1}^{m} M_i \times \left| \overline{a}_i u - \overline{c}_i \right| \right\},$$

$$\Pi_{\overline{A}} = \left\{ \overline{A} \,/\, \overline{\overline{A}}_{(H)} \leq \overline{A} \leq \overline{\overline{A}}_{(B)} \right\},$$

$$\Pi_{\overline{a}} = \left\{ \overline{a}_1 \,/\, \overline{\overline{a}}_{i(H)} \leq \overline{a}_i \leq \overline{\overline{a}}_{i(B)} \right\}, i = \overline{1,m},$$

$$\Pi_{\overline{C}} = \left\{ \overline{C} \,/\, \overline{\overline{C}}_{(H)} \leq \overline{C} \leq \overline{\overline{C}}_{(B)}, M_i, i = \overline{1,m} \right\},$$

$$\Pi_{\overline{B}} = \left\{ \overline{B} \,/\, \overline{\overline{B}}_{(H)} \leq \overline{B} \leq \overline{\overline{B}}_{(B)} \right\},$$

$$\Pi_u = \left\{ u \,/\, \overline{u}_{(H)} \leq u \leq \overline{u}_{(B)} \right\}.$$

Let's find the values of the parameters $\bar{A}, u, E_1, \bar{B}$ (present them as the results) that optimize the objective function, explore different dimensions of the problem. While studying such a problem (reducing it to discrete programming), for instance, using a genetic algorithm, the problem becomes high-dimensional. If to discretize each variable i, $(i = \overline{1, k_0})$ by values from the interval n_i of the problem, then such a problem leads to an overrun of options $P = \prod_{i=1}^{k_0} n_i$. As a result, it is possible to find an approximate solution (acceptable or invalid). To ease the burden of the large dimensionality of such a problem, it seems advisable to take measures at the stage of pre-optimization (analysis of the problem's solvability) and take into account the technological features of a particular problem at the modeling stage.

1.6 Hypotheses, assumptions and simplifications of a static ecological and economic model

The formulation of the inverse problem (to the classical direct one) of finding a mathematical model structure, the solution of which is the desired development highway of a static ecological and economic model, requires some additional refinements.

In particular, to outline the limitations in the construction of a given development highway:

A. The principle of "small steps" of exit.

The development highway can be considered in a global and local context. The global context defines the pipeline as a promising final benchmark for the state of the system as a result of organizational innovations and changes in general. It can be considered the result of a series of local step changes over a certain period of time. The global resultant vector is mathematically the sum of local trunk vectors of desired system states.

The local context focuses on the formation of a change vector and finding reasonable reorganization measures to refine the model within a limited period of time, taking into account the technological capabilities of the system to change.

B. The principle of local ("small") changes in the solution and the highway vector.

The parallelepiped of possible changes and the length of the change vector k_0 along the main line causing changes in the model, both in the technological matrix and in the constraint vector (blocks of technological matrices) are assumed to be local. This is due to the fact that the boundaries of the variables and the gradient can be considered as the result of a certain linearization of the nonlinear problem and the correctness of the nonlinearity representation by a linear approximation can be correct only in the neighborhood (local). Through the latter, let's extract y_1 and y_2, C.

C. The principle of local ("small") changes in model elements.

In terms of manufacturability, structural reorganization should be carried out within certain limited time frames, carried out evolutionarily, accompanied by small quantitative changes, taking into account the technological capabilities of the system.

D. The principle of correctness and conditionality of the system in case of changes in the model elements.

It is known that for correct, well-conditioned linear systems in modeling, small changes in the model correspond to small changes in the initial parameters of the system solutions. For instance, the conditionality number $M_A = \|A\| \times \|A^{-1}\|$ relates changes in the constraint vector in the model and the solutions of the problem.

While working with incorrect systems, disproportionate changes in the system are observed. Moreover, small quantitative changes in the model can be accompanied by significant qualitative changes in the output parameters. The nondegeneracy of the constraint matrix is violated, the rank of the technological matrix "falls", etc., and the uniqueness of the solution is not met. The correction (reorganization of the system) of constraints is considered as a multi-stage procedure of small changes, which is caused by the corresponding small changes in the main line.

It is necessary (provided that the system is correct) that small changes in the local vector of the development highway will cause small changes in the model elements such as the technological matrix, the constraint vector.

E. The principle of ergativity.

It is assumed that there is a decision-maker (DM) in the decision-making loop of the ecological and economic system.

The DM forms the resulting global development highway and its local components at different stages. The resultant highway (as a vector) is the sum of local highway vectors at the reorganization steps. The reorganization structure corresponding to the local trunk vector is determined by one of the mathematical methods (e.g., method of basis matrices).

The DM also forms the neighborhood of the solution change in the neighborhood (parallelepiped). Subsequently, mathematical procedures form an adequate length of the trunk vector. The DM can define the backbone vector as a certain cone with the points in the current state of the system. The DM can also eliminate ambiguities in the description of individual components.

F. The principle of "proximity" of individual elements of the model and the development pipeline.

It is evident that, the smaller the difference (proximity) of the main lines to, for instance, the product (cone of tolerance), the smaller the changes in the inverse matrix when replacing the column-item of the inverse matrix with the vector-column of the

main line. If the system is correct, small changes in the inverse matrix will cause small changes in the direct matrix, i.e., the constraint matrix of model (1.2).

If the equation $u = u_0 + k_0 e_k$ of the cone product (acceptable), and is the equation $\bar{u} = u_0 + k_0 e_l$ of the highway (desired), then it is possible to write $\bar{u} - u = k_0(e_l - e_k)$, $\|\bar{u} - u\| = |k_0| \|(e_l - e_k)\|$. That is, for "small differences", small deviations of the solutions and vectors of the generating equation from the main line are associated. The "small" deviations of the product and the backbone vector during the substitution operation in the inverse matrix of the vector-column of the product vector with the backbone vector will have small changes. If the system is correct, it will cause small changes in the direct matrix, as well as their norms. The latter, the direct matrix, is the forming matrix of system (1.2). With "good" conditioning (the product of the norms of the new direct and inverse matrices will be a small number).

1.7 Technology of reorganization in a static ecological and economic model to achieve the values of main indicators

Developing the reorganization technology, the method of basis matrices (MBM) was applied. This is a method of the type of simplex-methods which is aimed at analyzing and solving the above problems, both direct and inverse. It should be noted that these are primarily problems of linear systems analysis, in particular, linear algebraic equation system (LAES), and are basic ones mentioned by V. Kudin, V. Onotskyi, A. Al-Ammouri, L. Shkvarchuk in conducting more complex studies and generalizations. As a rule, modern computers use standard types of integers which size does not exceed 64 bytes [10].

This hardware limitation was overcome by software, namely, by developing our own data type in the form of a special Longnum library in C++ using the Standard Template Library (STL). The software implementation was developed to perform calculations using the MBM and Gauss, i.e., long arithmetic for models with rational elements was used. The algorithms and computer implementation of Gauss-type methods and artificial basis matrices (a variant of the basis matrix method) in Matlab and Visual C++ environments using the technology of accurate computation of method elements, primarily for ill-conditioned systems of different dimensions, are proposed.

It should be noted that the MBM has been thoroughly tested (as a "solver" of a system of linear algebraic equations), in particular, for geohydrodynamics problems. In particular, procedures for finding and refining the solution for models of geohydrodynamic problems were developed on the basis of the MBM; software for

computational experimentation was developed for typical models; the reliability was checked, and the main parameters (solution accuracy, solution time) of the MBM computational results were compared with the Gauss, SVD, and Greville methods depending on the number of conditionality.

The aforementioned problems are characterized by poor conditionality, bad completeness of the constraint matrix (sparsity), large dimensionality, and certain structural features of the constraint matrix. A number of these properties are also inherent in mathematical models (1.2) and (1.3), both for forward and inverse problems.

Nowadays the study of MBM algorithms has been neglected in the analysis of inverse problems. Thus, the effect of the implementation of the desired master values (the vector of values of the components of the output parameters) on the structural properties of the matrix model (in a linear system) on the various components of the model. For the direct problem, such studies (to analyze the impact of such changes in the model elements) on the properties of a linear system (without solving the problem) were conducted by V. Kudin, A. Onyshchenko, I. Onyshchenko – element, row, column of the constraint matrix, etc. [11].

The scheme of the method of artificial basis matrices is a specification of the MBM to analyze the properties of linear systems of equations and inequalities, in particular, with a square constraint matrix. The basis of the proposed method of artificial basis matrices and its corresponding algorithms is the idea of a basis matrix formed by linearly independent rows of the constraint matrix. The process of iterative splicing (replacement) of the constraints of the auxiliary system with the relaxed constraints of the main system is carried out.

The square matrix mentioned by V. Kudin, V. Onotskyi, A. Al-Ammouri, L. Shkvarchuk (in general, a submatrix) $A_b = A = \|a_{ij}\|, i = \overline{1,m}, j = \overline{1,m}$, consisting of m linearly independent row-normals $a_i = [a_{i1}, a_{i2}, ..., a_{im}], i = \overline{1,m}$, will be called an artificial basis, and a solution $u_0 = (u_{01}, u_{02}, ..., u_{0m})^T$ of the corresponding system of equations $A_b u = C^0$, where $C^0 = (c_1, c_2, ..., c_m)^T$ is an artificial basis [10].

Definition 1. In the general case, let's understand the highway as a cone formed by the tip (solution vector) and the vector (set of solution vectors) with nonnegative components, which indicates the "desired membership set" of solutions (values of the "system output") of the directionally reconfigured ecological and economic system (as a solution to the inverse problem).

Definition 2. The reconfigured system is a mathematical model of the form (1.3) obtained as a result of the implementation of the main guidelines for sustainable growth at time t (step of change).

Definition 3. In this formulation, the mainline is defined by the vector $u = u_0 + t \times e_{,i}$, $t > 0$, where $e_{,i}$ is the mainline vector. It is assumed that the backbone vector

should not "fall out" of the parallelepiped of variables centered at the point u_0 with its changes

$$\Pi_{u_0} = \left\{ u = (u_1, u_2, \ldots u_m)^T / u_{0i(H)} \leq u_{0i} \leq u_{0i(B)}, i \in I \right\}.$$

Let's suppose, e_{ri} are the elements of the matrix $A^{-1} = \|e_{ij}\|$, $i = \overline{1,m}$, $j = \overline{1,m}$, composed of m linearly independent column vectors $e_j = (e_{1j}, e_{2j}, \ldots, e_{mj})$, $j = \overline{1,m}$ inverse to $A = A_b$; $\alpha_{.i} = (\alpha_{i1}, \alpha_{i2}, \ldots, \alpha_{mj})$, $j = \overline{1,m}$ is the development vector of the constraint vector α_i along the rows of the basis matrix $A = A_b$ $(\alpha_{i.} = \alpha_i \times A^{-1})$; $\beta_{.l} = (\beta_{l1}, \beta_{l2}, \ldots, \beta_{lm})$ – column expansion vector of the backbone vector $e_{.l} = (e_{1l}, e_{2l}, \ldots, e_{ml})$ along the columns of the matrix A^{-1} $(\beta_{.l} = A \times e_{.l})$; $\Delta_r = \alpha_r u_0 - c_r A^{-1}$ $(\beta_{.l} = A \times e_{.l})$; $\Delta_r = \alpha_r u_0 - c_r$ – the invariance of the r constraint (1.1) at the vertex u_0. All the introduced elements in the new basis matrix $\overline{A}_b$, which differ from A_b by one row, will be marked with a dash from above. The ideology of the simplex methods (including the method of basis matrices) establishes the relations between the direct and inverse matrices when replacing one row (column) with another, subject to the condition of non-degeneracy (support) of the newly formed matrices. It should be noted that the category (names) of the direct and inverse matrices are relative in the context of a particular application. In particular, for the direct and inverse matrices, the following relations hold

$$A_b \times A_b^{-1} = E\left(\left(A_b^{-1} \right)^T \times A_b^T \right) = E.$$

In the left formula, let's consider A_b to be a direct matrix and the second one to be an inverse matrix to it and in the second formula (in parentheses), $(A_b^{-1})^T$ is a direct matrix and the second one A_b^T an inverse matrix (as if it were the other way around). Here, the matrix where the simplex iteration of row (column) substitution is performed is a direct matrix, and the matrix determined as a result of the substitution will be the inverse (to this direct matrix).

According to Theorem 1 (numerical mathematics), the relationship between the coefficients of the development of the constraint normals, elements of the inverse matrices, basis solutions, and constraint disjunctions in two adjacent basis matrices is established. A scheme for determining the rank of the system's constraint matrix and the solution of the system of equations can be built based on them by successive changes in the basis matrices and the corresponding artificial solutions.

It is known that the properties of a LAES of type (1.1) have connections with the corresponding SLAN (system of linear algebraic inequalities). Namely

$$Au \leq \overline{C}. \tag{1.4}$$

The linear algebraic inequality system (LAIS) (1.4) is formed by the intersection of nonpositive half-spaces generated by the hyperplanes of constraints (1.2). The influence of changes in the model is usually studied through a direct constraint matrix in practical problem formulations (e.g., the problem of the influence of perturbations in the elements of the constraint matrix on the properties of the solution). However, it happens that the problem is posed vice versa, according to the given properties of the problem solution, it is necessary to recreate a model (reconfigure) that will have the given solution properties. In such cases, the main emphasis of analyzing the impact of changes on the solution is studying the properties of the already inverse matrix.

For instance, there is a desired vector of a new solution that has certain properties (in relations) with the cone components. In this case, it is natural to "start from" the structure of the inverse matrix (columnar) in the analysis. If to represent the direct and inverse matrices in a transposed form, it follows that the columnar matrix of the derivatives (inverse) becomes a direct (ordinal) matrix. The analysis of changes in such a matrix can be carried out on the basis of the ordinal strategy of the MBM, in particular, to determine the properties of the inverse matrix to it (i.e., the original model).

It is evident that the smaller the difference (proximity) of the trunk vector to the product vector, the smaller the changes in the inverse matrix when replacing the column – the product of the inverse matrix with the trunk vector. Assuming the correctness of the system, small changes in the inverse matrix will cause small changes in the direct matrix, i.e., the constraint matrix of the model (reconfigured system).

1.8 Conception of SDGs and algorithm for building directed changes

As for the basic statements and algorithm for building directed changes (development highways) based on the given initial benchmarks, for solving the inverse problem, let's further apply the provisions of the method of basis matrices and its algorithm, in particular, the formulas for recalculating the elements of the method. Assuming that the direct matrix of the simplex iteration of the basis matrix method is $(A_b^{-1})^T$, and the inverse matrix is A_b^T $((A_b^{-1})^T \times A_b^T) = E)$ when the k position of the direct matrix e_l (already a row) is replaced by the main line e_l (whose development vector is already $\beta_{.l} = (\beta_{1l}, \beta_{2l}, ..., \beta_{ml})$, $\beta_{kl} \neq 0$ (support condition), the columns of the newly formed inverse matrix $\bar{A}_b^T$ are given by the relations

$$a_k^{-T} = \frac{a_{k.}^T}{\beta_{kl}}, a_k^{-T} = a_{i.}^T - \frac{a_{k.}^T}{\beta_{kl}} \beta_{il}, \beta_{kl} \neq 0, i \neq k, i \in I. \tag{1.5}$$

For the resulting matrices, either $\overline{(A_b^{-1})^T} \times \overline{A}_b^T = E$ or $\overline{A}_b \times \overline{A}_b^{-1} = E$ will be performed. In this case, the iteration is performed by replacing the product vector, for instance, with the backbone vector.

The closeness of the product and backbone vectors (angle γ) can be "measured" by the ratio

$$\cos(\overbrace{e_{.k},e_{.l}}^{\gamma}) = \frac{(e_{.k}^T, e_{.l}^T)}{\left\|e_{.k}^T\right\| \times \left\|e_{.l}^T\right\|}, \min_{i \in I} \cos(\overbrace{e_{.k},e_{.l}}^{\gamma}), k = \underbrace{\arg\min}_{i \in I} |\cos(\overbrace{e_{.k},e_{.l}}^{\gamma})|,$$

where the index k is the column number of the candidate vector to be replaced by the backbone vector.

It is possible to write that $\overline{u} - u = k_0 (e_{.l} - e_{.k}), \|\overline{u} - u\| = |k_0| \| (e_{.l} - e_{.k}) \|$. It follows that for "small differences", "small" deviations of the solutions and vectors of the product from the backbone are associated with "large differences" and for "large differences", large deviations are associated with the backbone. "Small" deviations of the product and the backbone vector in the case of the replacement operation in the inverse matrix of the product vector by the backbone vector will "give" small changes and vice versa. If the system is correct, this will cause small changes in the direct matrix, as well as their norms (the reconfigured system). The latter (direct matrix) is the forming matrix of the system (1.3). In particular, with "good" conditionality (the product of the new direct and inverse matrices will be a small number). It is not difficult to see that the recalculation of the elements of the inverse matrix (already in the inverse problem) corresponds to one iteration of the MBM, i.e., to the solution when the backbone vector is introduced into the basis matrix.

For each constraint (1.3), let's introduce quantities

$$L_i = \left[L_{i(H)}, L_{i(B)} \right], R_i = \left[c_{i(H)}, c_{i(B)} \right], \overline{u} = u_0 + k_0 \times e_{.l},$$

where

$$L_{i(H)} = \min\left\{ (a_i, u) / a_{i(H)} \leq a_i \leq a_{i(B)}, u_{(H)} \leq u \leq u_{(B)}, i \in I \right\},$$

$$L_{i(B)} = \max\left\{ (a_i, u) / a_{i(H)} \leq a_i \leq a_{i(B)}, u_{(H)} \leq u \leq u_{(B)}, i \in I \right\}.$$

Proposition 1. A sufficient condition for $\overline{u} = \Pi_u$ the absence of the corresponding matrix and constraint vector from the set of tolerances ($A \in \Pi_{\overline{A}}, C \in \Pi_{\overline{C}}$) for a given developmental path is the fulfillment of the following condition $\exists i_0 \in I, L_{i_0} \cap R_{i_0} = \varnothing$.

Proof. The introduced values ($\forall i \in I, L_i, R_i$) define, respectively, the region of values of the left side of each equation and the right side of (1.3). The existence of at least one equation from the system for which the intersection of the regions of values is an empty set indicates the insolvability of the equation and the entire system of equations. That is, there will be no matrix or vector of constraints forming a system of type (1.3) whose solution corresponds to the developmental highway.

Proposition 2. A necessary condition for $\bar{u} \in \Pi_u$ the existence of a matrix and a vector of constraints from the set of tolerances ($A \in \Pi_{\bar{A}}, C \in \Pi_{\bar{C}}$) for a given developmental mainline is the fulfillment of the condition $\forall i \in I, L_i \cap R_i \neq \varnothing$.

Proof. The existence of a non-empty intersection of the regions of values for all equations from the system (for the previously introduced value $\forall i \in I, L_i, R_i$ indicates the solvability of the equation and the entire system of equations as a whole. That is, there is room to continue searching for the matrix and constraint vector forming a system of type (1.3), the solution of which will correspond to the development highway.

In general, the fulfillment of the performance conditions (model properties) implies the positivity of the model elements when transformed to the form (1.2). If the condition of preserving the positivity of the model elements when transformed to the system (1.2) is met, a number of properties are established for such a model.

Proposition 3. A necessary condition for preserving the positivity of the elements of the reconfigured model (1.3) under transformation in the direction of the trunk $e_{\cdot l}$ (which development vector $\beta_{\cdot l} = (\beta_{1l}, \beta_{2l}, ..., \beta_{ml})$) is the existence of an index k such that $\beta_{kl} > 0$.

Proof. According to the formula (for k) of the "new" column $a_k^{-T} = a_k^T / \beta_{kl} > 0$ when the condition $\beta_{kl} > 0$ is fulfilled. Since, $a_k^T > 0$ under the assumption of the positivity of the elements of the model (1.2).

Corollary 1. A sufficient condition for the absence of a reconfigured model (1.3) with positive elements when transformed in the direction of the trunk $e_{\cdot l}$ (which development vector $\beta_{\cdot l} = (\beta_{1l}, \beta_{2l}, ..., \beta_{ml})$) is that $\beta_{il} < 0, i = \overline{1, m}$.

Proof. According to the transformation formula, if the k column is chosen arbitrarily, the "new" column (leading) will be $a_k^{-T} = a_k^T / \beta_{lk} < 0$ when the condition $\beta_{kl} < 0$, $a_k > 0$ is fulfilled. Since, $a_{k\cdot}^T > 0$ under the assumption of the positivity of the elements of the initial model (1.2).

Corollary 2. A sufficient condition for the reconfigured model (1.3) to retain positive elements when transformed in the direction of the trunk line $e_{\cdot l}$ (whose development vector $\beta_{\cdot l} = (\beta_{1l}, \beta_{2l}, ..., \beta_{ml})$) is the existence of an index k such that $\beta_{kl} > 0$ and $\beta_{il} < 0, i = \overline{1, m} (i \neq k)$.

Next, let's present an algorithm for constructing a system of type (1.2) for a given system, taking into account the bilateral constraints on variables and without taking into account the restriction on the positivity of the elements of the model of the original reconfigured system as a result of transformations caused by the main changes, i.e., solving the inverse problem.

Algorithm.

Preparatory step. Let u_0, $A = A_b$ (its rows a_i, $i = \overline{1,m}$, A_b^{-1} (its columns $e_{.i}$, $i = \overline{1,m}$ for LAES (1.2) representing the ecological and economic model) be known. Let's form the boundaries of permissible changes in the initial parameters of the model as a parallelepiped of the form

$$\Pi_{u_0} = \left\{ u = (u_1, u_2, \ldots u_m)^T / u_{0i(H)} \leq u_{0i} \leq u_{0i(B)}, i \in I \right\}, u_0 \in \Pi_{u_0}.$$

Step 1. Decision makers (DMs) determine the column vector of the highway – $e_{.I}$, i.e. the desired vectors of changes in the initial system of parameters in the form of the vector $u = u_0 + t \times e_{.I}, t > 0$.

Step 2. Find the development of $e_{.I}$ – the column vector of the main line by columns A_b^{-1} according to the ratios $(\beta_{.I} = A \times e_{.I})$.

Step 3. Check the support condition $\beta_{kl} \neq 0$ when performing the operation of replacing column k $(e_{.k})$ with the trunk vector $e_{.I}$ in the matrix A_b^{-1} (form $(A_b^{-1})^T$). This is analogous to the operation of replacing the row k $(e_{.k}^T)$ with the backbone vector-row (transposed $(e_{.I}^T)$) in the matrix, forming $(A_b^{-1})^T$.

Step 4. Form the columns of the newly formed (inverse of A_b^T!) matrix, i.e., the matrix $\bar{A}$ by the ratios

$$a_k^{-T} = \frac{a_{k.}^T}{\beta_{kl}}, a_{i.}^{-T} = a_{i.}^T - \frac{a_{k.}^T}{\beta_{kl}} \beta_{il}, \beta_{kl} \neq 0, i \neq k, i \in I.$$

Step 5. Find the values $k_0 = \min\limits_{i \in I} \left(\dfrac{u_{i(B)} - u_{i(H)}}{2} \right)$ and form $\bar{u} = u_0 + k_0 \times e_{.I}$.

Step 6. Putting the values $\bar{C} = \bar{A}(u_0 + k_0 e_{.I})$.

The final step. Determining the amount of changes in the model elements during the reorganization of the system using the formulas

$$C' = \bar{C} - C, A' = \bar{A} - A.$$

Steps 1–4 applying the MBM relations (one iteration) to the inverse problem – finding the constraint matrix of the corresponding development pipeline.

Step 5 adjusting the step size of the backbone vector in accordance with the constraints on the variables.

Step 6 is the formation of a constraint vector corresponding to the development pipeline.

At the final step, let's determine the magnitude of changes (deviations) in the newly formed structure (matrix and constraint vector) that implement the development pipeline relative to the initial values of the system elements.

The above steps generally define the procedure for solving the inverse problem. The proposed algorithm finds a matrix and a vector of constraints, i.e., the components of the mathematical model whose solution is realized by the development highway. The core of the algorithm of the inverse problem (construction of the development highway) is the iteration of the additional solution (refinement) of the MBM.

The inverse problem (construction of a static problem development highway) is reduced to the use of linear systems tools (direct problems) that have been tested on other classes of problems. The computational complexity of the steps of the procedure for solving the inverse problem (building a static problem development highway) is correlated with the iteration of the MBM (forward problem).

Hence, a sustainable development involves improving of the quality of life of the entire world's population without increasing the use of natural resources to a level that exceeds the capacity of the Earth as an ecological system. Efforts for creating a sustainable way of life involve a comprehensive approach to activities in three key areas (**Fig. 1.5**), such as:

– economic growth and equity – applying a comprehensive approach to stimulate long-term economic growth;

– conservation of natural resources and environmental protection – finding economically viable solutions to the problem of reducing resource consumption and stopping environmental pollution;

– social development – meeting the needs of people for work, food, education, energy, healthcare, ensuring cultural and social diversity and the rights of workers, providing opportunities for all members of society to participate in decision-making that affects their future.

In our opinion, the economic approach to the concept involves the optimal use of limited resources and the use of environmentally friendly, nature conservation and energy saving technologies, as well as technologies for recycling and emission destruction. However, when deciding what kind of capital should be preserved and to what extent different types of capital are interchangeable, as well as when valuing assets, especially environmental resources, there are problems of proper interpretation and calculation.
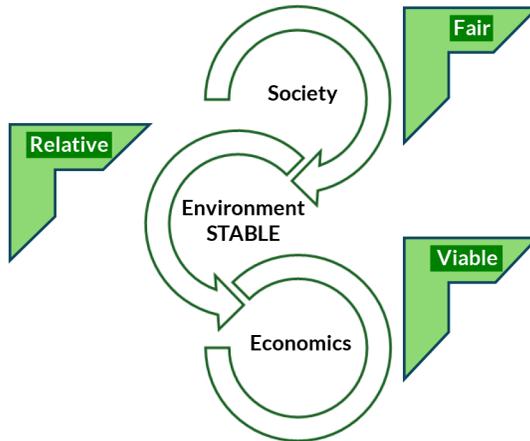
Fig. 1.5 Conception of Sustainable Development Goals
Source: compiled by the authors

Two types of sustainability have emerged: 1) weak, when it comes to time-decreasing natural and productive capital, and 2) strong, when, in addition to time-decreasing natural capital and a portion of the profit from the sale of non-renewable resources, it is necessary to increase the value of renewable natural capital.

1.9 Conclusions

It is established that the solution of the inverse problem of finding the structure of a mathematical model (for a given developmental path) in terms of computational volume corresponds to one iteration of the MBM solution. A number of statements are substantiated; in particular, the conditions for the existence of the corresponding matrix, constraint vector and their positivity for the development mainline. The solution of the master problem in the course of modeling provides feedback in the system. This is important for analyzing the impact of different "scenarios" of sustainable development (the main line) on the constraint matrix and the constraint vector of the reconfigured ecological and economic model. An algorithm is proposed, which in essence is an iterative procedure for refining the initial model by conducting similar iterations of the MBM. That is, finding a reconfigured (directionally changed) system as a result of the impact of mainstream changes to the inverse problem.

Since it has been established that the MBM can be applied to both the direct and inverse problems, the results of the experiments and the comparison of computational schemes for linear systems conducted earlier can be taken into account in further applications. The computational experiment on the model example shows that the main computational load of the algorithm is the iteration of the MBM solving.

The social component of sustainable development is human-centered and aims to maintain the stability of social and cultural systems. An important aspect of this approach is the fair distribution of benefits. The society will have to create a more efficient decision-making system to achieve sustainable development. It is important to get not only internal but also intergenerational justice. From an environmental perspective, sustainable development must ensure the integrity of biological and physical natural systems. Of particular importance it is the viability of ecosystems, which the global stability of the entire biosphere depends on. The particular attention is paid to the preservation of self-healing and dynamic adaptation of such systems to changes, rather than preserving them in some ideal static state.

Reconciling these different perspectives and implementing specific measures is a rather complex task, as all three elements of sustainable development must be considered in a balanced manner. The mechanisms of interaction between these three concepts are also important. The economic and social elements interact to generate new challenges such as achieving equity, social guarantees, the right to work, and providing assistance to the poor. The interplay of the economic and environmental elements gives rise to new considerations regarding the valuation and inclusion of external environmental impacts and efficient environmental management in economic reporting. The linkage of social and environmental elements arouses interest in such issues as respect for the rights of future generations, public participation in decision-making, and control over climate change.

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